SYNTHESIS OF MULTI-LINK MECHANISMS FOR DYNAMIC SPECIFICATIONS

BY

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My heartfelt thanks are expressed also to my wife, Marilyn, for her tolerance and support.

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SYNTHESIS OF MULTI-LINK MECHANISMS FOR DYNAMIC SPECIFICATIONS

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Chairman: Dr. Delbert Tesar
Major Department: Mechanical Engineering

A theory is presented for the exact synthesis of five-, six- and higher-link, planar mechanisms for five multiply separated position dynamic specifications (the class of prescribed conditions for synthesis consisting of inertia forces and torques, momenta or combinations of positions and time derivatives up to the fourth order, i.e., velocity, acceleration, jerk and jounce). Also, certain previously unsolved geometric synthesis problems are solved utilizing this theory.

A generalized computer program, MECSYN, is presented for the synthesis of multi-link, planar mechanisms to allow practical use of the theory above in addition to the usual synthesis for geometric specifications. MECSYN is a closed form, noniterative program which uses a modular approach employing four tools of synthesis (constraint link synthesis, inversion, angular cognates and path cognates) for all possible cases of four and five multiply separated positions. Other features of MECSYN are complete mechanism analysis for the specified precision conditions, link length ratios, Grashof's criteria, straight line motion synthesis, and function generation for four-, five- and six-link mechanisms.

Examples are presented for four-, geared five- and six-link mechanisms, including two practical examples. A Stephenson 3 and a Stephenson 2 six-link mechanism is designed to transfer an object from a continuously moving conveyor to a continuously rotating turret so that impact forces are eliminated. A Stephenson 3 mechanism is designed to allow automatic torch or bandsaw cutting of large, low pitch (Pd = 3/8) gear teeth.

CHAPTER I

INTRODUCTION

A. A Short History of "Kinematic" Synthesis

In recent years, many researchers have investigated the problem of approximating a function or the coplanar motion of a body with a mechanism. The impetus for this research was the advent of the digital computer. Although some of these solutions had been obtained by graphical means in previous years, many required enormous efforts and painstaking attention to accuracy to produce mechanisms for the maximum number of solvable specified conditions. Also, generation of a large number of general mechanisms to allow selection on the basis of preferred mechanism properties was usually impractical.

Most of this research was concentrated on synthesizing a four-bar mechanism such that a point in the coupler plane passed through three to five positions while the coupler angle had prescribed rotations between these positions. This is referred to in the literature as finitely separated positions (FSP), while specification of geometric derivatives for the path point is referred to as infinitesimally separated positions (ISP). No derivative conditions were specified for the coupler rotation angle since it was the independent geometric parameter for the specified path point derivatives.

This type of synthesis has been known in the literature as "kinematic" synthesis when, in fact, it has not included specifications for kinematics in the fullest definition of the term (such as velocity, acceleration, jerk and jounce). The objective was to synthesize mechanisms to satisfy purely geometric properties (such as the tangent, curvature, inflection, etc. of a given function). The result of this type of synthesis is a mechanism which satisfies certain geometric properties regardless of how it is driven (i.e., input velocity, acceleration, jerk, etc.), and will be referred to hereafter as geometric synthesis.

Five finitely separated position synthesis of four-bars was done graphically by Burmester [1] in 1888. This problem was solved analytically by Primrose, Freudenstein and Sandor [2] in 1964 and Bottema [3] also in 1968. Graphical studies on five infinitesimally separated positions were given by Mueller (1895, [4]), Allievi (1895, [5]), Krause (1920, [6]), Wolford (1960, [7]) and Veldkamp (1963, [8]). A unified theory for treating all seven combinations of finitely and infinitesimally separated positions was developed by Tesar et al. (1968, [9]), and is referred to as five multiply separated positions (MSP).

Frequently, the number of free parameters for synthesis available in four-bar mechanisms is not sufficient to adequately solve the problem at hand. In this case, the next two classes of constrained planar mechanisms that can be synthesized are geared five-link and six-link mechanisms. Dynamic specification synthesis as presented here would not be possible without the capability to synthesize six-link mechanisms due to the large number of parameters required.

Function generation for the Watt 2 (two four-bars in series) has been investigated several times. Solutions have been presented for five finitely separated precision points in closed form and numerical techniques have been presented for up to nine precision points by authors such as Kramer and Sandor [10], Levitskii [11] and McLarnan [12].* Hain presents several synthesis techniques for six-bar linkages [13]. However, all the solutions are obtained by graphical techniques and vary from two- to five-position synthesis.

A uniform theory for the solution of more than 35 distinct six-link synthesis problems for five FSP and in many cases five MSP was presented by the author [14] and the author and Tesar [15] in 1971. Four basic tools of synthesis were utilized (curvature transform, inversion, path cognates and angular cognates) of which only the first two had been used extensively in the past. This work allows the solution of the dynamic specification synthesis problem considered in Chapter II by providing the necessary number of free synthesis parameters available in terms of six-link mechanisms.

Nine precision point synthesis of geared five-bar mechanisms was solved numerically by Roth [16] in 1962 and geared five-bar function generation has been developed by Oleksa and Tesar [17] and independently

---

*It should be noted that the Watt 2 mechanism has no coupler motion of higher order than a four-bar (sixth order algebraic curve), while other six-link mechanisms have up to fourteenth order algebraic curve coupler paths. As a result the Watt 2 is the simplest to synthesize and has restrictions similar to a four-bar. One such restriction is that for higher than five precision points, the existence of a solution is not guaranteed.

by Sandor and Erdman [18] in 1970. The author solved the remaining problem of five-position coplanar synthesis for geared five-bars also in [14] and with Tesar [19] in 1971.

All of the preceding references solve only the geometric synthesis problem. The objective here is to synthesize mechanisms for dynamic specifications as will be described in the following section.

B. Scope of This Study

Preliminary work in the synthesis of mechanisms to satisfy dynamic specifications was done by Eschenbach and Tesar [20,21] and Eschenbach [22] for a four-bar mechanism by a different method than will be presented here. Also, emphasis was placed on selection of constraints based on force transmission angle of a four-bar. Eschenbach [22] concluded: "Thus it is impossible to coordinate the complete dynamic state of a simple crank and body beyond the second order acceleration state," in the context of four-bar mechanisms. The first objective of this study will be to give closed form solutions for the coordination of a simple crank input with the complete dynamic specification of a body for seven possible combinations of specifications (described in Chapter II) including fourth order time derivatives. Because of the previously noted limitation in terms of available parameters, this must be done by synthesis of geared five-link and six-link mechanisms. The objectives may be stated as follows:

1. Presentation of a theory for the exact synthesis of five-, six- and multi-link mechanisms for dynamic specifications (Chapter II). The term dynamic specifications is used to mean the class of prescribed conditions for synthesis consisting of

inertia forces and torques, momenta or combinations of position and time derivatives up to fourth order (i.e., velocity, acceleration, jerk and jounce). This does not include all possible problem definitions where a general applied force or torque is specified. The maximum number of conditions which will allow exact solutions are specified for each link (i.e., five-position problem). Also, certain previously unsolved geometric synthesis problems are now solvable.

2. Development of a generalized computer program (Chapter V) for the synthesis of four-, five-, six- and multi-link mechanisms to allow practical use of the theory listed above in addition to synthesis for purely geometric specifications. A modular approach is used employing the four tools of synthesis employed by the author in six-link synthesis [14] (constraint link synthesis, inversion, angular cognates and path cognates). Other synthesis tools useful to designers are included such as analysis of the resulting mechanisms at the specified precision conditions, link length ratios and Grashof's mobility criteria.
3. Application of 1 and 2 in the solution of examples for four-, five- and six-link mechanisms (Chapter III) including two practical examples (Chapter IV).

C. Six-Link Mechanisms

A constrained closed chain is one in which the motion of each link is uniquely determined with respect to the other links. This study will deal only with constrained, planar mechanisms having revolute, prismatic or gear pairs (joints). Most mechanisms can serve a useful

purpose only if the motion is predetermined, given a single input. On occasion mechanisms are desired which require two or more inputs to produce a certain motion such as control devices or differential mechanisms. These are not constrained mechanisms but may be synthesized by the methods presented here.

Due to the work of Grübler and others, it is possible to determine the number of degrees of freedom of a planar mechanism from the number of links and the number of joints alone,

$$F = 3(n-1) - 2g - h,$$

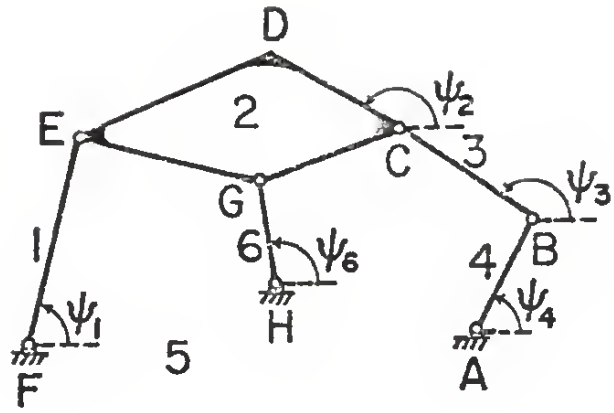
where F is the number of degrees of freedom, n is the number of links including the fixed link, g is the number of lower pairs and h is the number of higher pairs. Thus for a closed planar turning pair chain to be constrained (F = 1),

$$g = (3/2)n - 2,$$

and for closed six-link turning pair mechanisms the number of turning pairs must be seven.

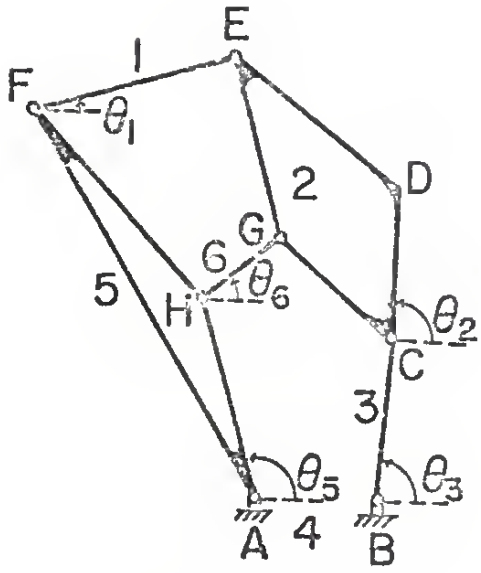
Grübler also proved that the minimum number of binary links (having two kinematic pairs) in any constrained chain is four and the greatest number of pairs by which any link of the chain can be joined is n/2 (n--total number of links). Thus a six-bar chain (constrained) must have at least four binary links, cannot have any quaternary links (having four kinematic pairs) and can have at most two ternary links.

There are two chains having four binary and two ternary links. The Stephenson chain Fig. (1-1) has a binary link separating the two



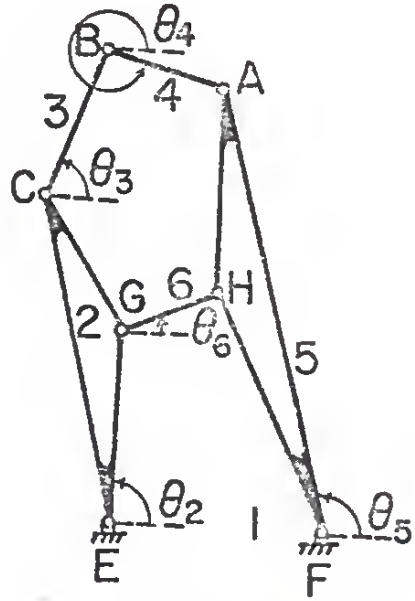
Stephenson 3

a.



Stephenson 2

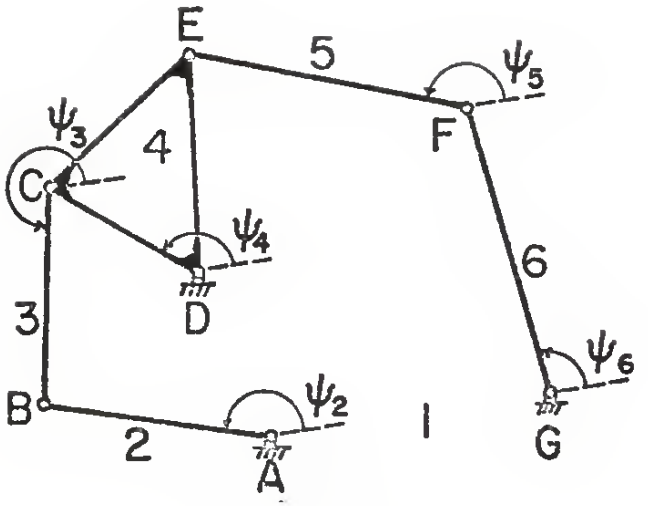
b.



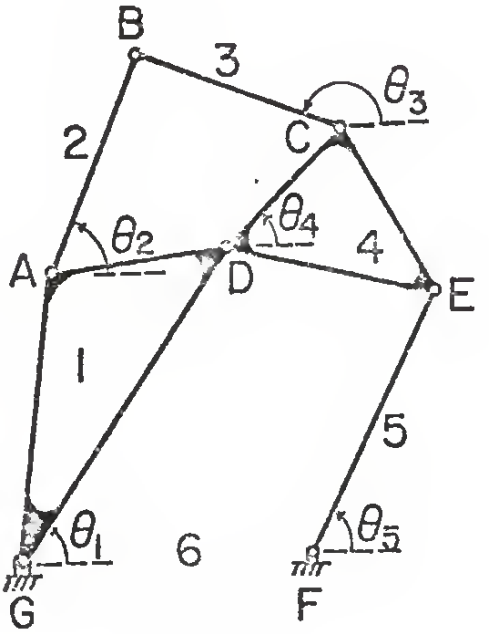
Stephenson I

c.

Figure 1-1 Stephenson Six-Link Chain



a. Watt 2



b. Watt I

Figure 1-2 Watt Six-Link Chain

ternary links. The Watt chain Fig. (1-2) has two ternary links joined together. There is one chain having five binary links and one ternary link requiring one double joint (two turning pairs having a common axis of revolution). Six binary links having two double joints make up the last six-link, constrained chain. The last two chains can be described as special cases of the Stephenson and Watt chains. They have fewer free parameters for synthesis and will not be separately considered here.

All possible distinct inversions of the Stephenson and Watt chains are illustrated in Figs. (1-1, 1-2). Some inversions of a chain are equivalent due to the conceptual symmetry of the chain.

There are three distinct inversions of the Stephenson chain. Fixing binary link 1 or 6, Fig. (1-1c) produces a Stephenson 1 mechanism. Fixing link 3 or 4 gives a Stephenson 2 mechanism, Fig. (1-1b). Fixing either ternary (2 or 5) yields a Stephenson 3 mechanism, Fig. (1-1a).

Fixing any binary link of the Watt chain produces a Watt 1 mechanism, Fig. (1-2b). Fixing either ternary link gives a Watt 2 mechanism, Fig. (1-2a) which is equivalent to a pair of four-bars in series (possessing a common link and fixed pivot of that link).

D. Four Tools of Synthesis

It will be shown that the objectives stated above can be achieved by the use of four tools of synthesis: constraint link synthesis, inversion, path cognates and angular cognates.

1. Constraint Link Synthesis

Constraint link synthesis has also been termed curvature synthesis from classical curvature theory. This is probably the most

important and most difficult aspect of the synthesis process. Given a set of desired specifications for a link or links, the objective is to find solutions for constraint links which will form a constrained mechanism to satisfy the specifications. Several different methods of constraint link synthesis have evolved for geometric synthesis; i.e., graphical, analytical and iterative numerical techniques. Although analytical solutions are the most desirable for many reasons (accuracy, computing time, etc.) they are the most difficult to formulate. Since geometric synthesis is not sufficient to design mechanisms for dynamic specifications, a new curvature transform must be developed. Chapter II presents a closed form analytical constraint link synthesis formulation for dynamic specifications also called the dynamic specifications curvature transform or the dynamic curvature transform. Solutions are found for the maximum number of specifications in terms of the number of parameters for the mechanisms treated.

In keeping with the closed form analytical nature of the constraint link synthesis presented here, all other synthesis tools are solved in closed form including the analysis (Chapter III). This allows the computer program, MECSYN (Chapter V) to be precisely verified and highly efficient.

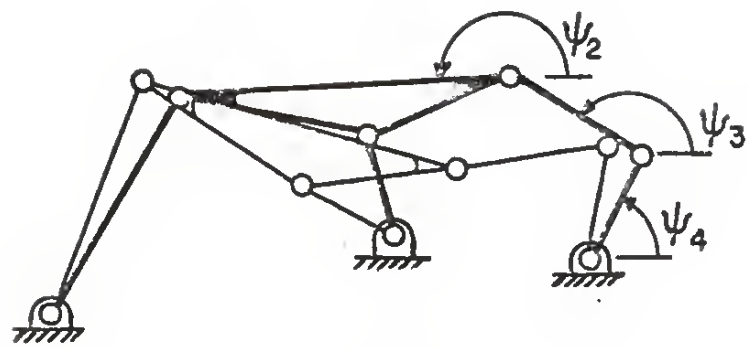
2. Inversion

A group of links connected together with no links rigidly fixed is called a kinematic chain. If a chain is arranged so that the motion of one link causes a definite motion of every other link relative to the first link, it is called a constrained chain. Motion of one link

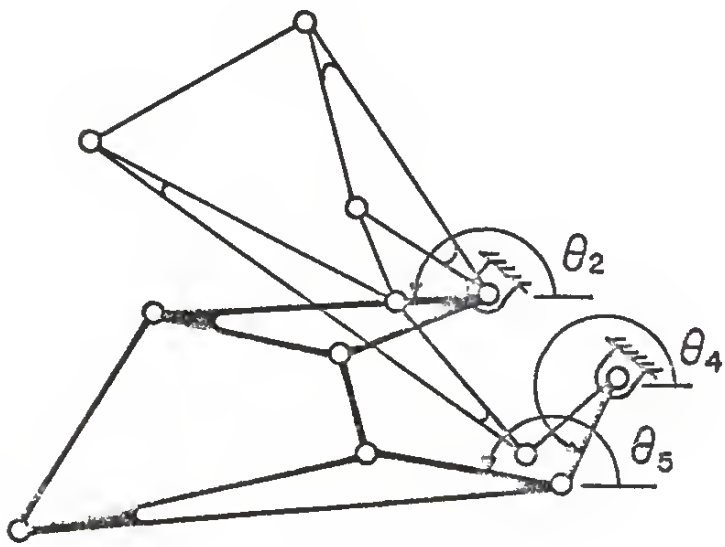
of the chain constrains the remaining links to execute determinate relative motions. A constrained chain does not produce any definite absolute motion. When one of the links is fixed, the relative motions of the chain with respect to that link become absolute. The chain becomes a mechanism. A constrained kinematic chain can be made into a mechanism in as many ways as it has links. Some of these mechanisms may be equivalent due to conceptual symmetry of the chain.

The significance of inversion is that the motions of all members of a chain with respect to each other, that is, their relative motions are independent of which member serves as the frame or fixed link. It will be shown that inversion is a useful tool for synthesis of multi-link mechanisms because it allows synthesis techniques for simple mechanisms (four-bar) to be extended to the design of more complex mechanisms. For example, each six-link chain has one inversion that can be synthesized with less difficulty than the others. These are the Stephenson 3, Fig. (1-1a) and the Watt 2, Fig. (1-2a). Using inversion, specifications may be chosen, e.g., for a Stephenson 2 mechanism and converted to equivalent synthesis specifications for a Stephenson 3. Synthesis may then be carried out and the resulting Stephenson 3 mechanism reinverted to obtain the Stephenson 2 which satisfies the chosen specifications.

Although the equations of inversion involve only angular properties of links, it will be shown in Chapters III and IV how inversion may be used to synthesize mechanisms for specified path and angular properties. A Stephenson 3 mechanism is shown in Fig. (1-3a) for two positions



a.



b.

Figure 1-3 Inversion of a Stephenson Six-Link Mechanism

of the chain. The exact same chain is shown for the same two positions as a Stephenson 2 inversion in Fig. (1-3b). The angular positions and higher order time derivatives of links in these two inversions are related by

$$\begin{aligned}
\psi_{nj} &= \theta_{nj} - \theta_{5j} + \theta_{50} \\
\dot{\psi}_{nj} &= \dot{\theta}_{nj} - \dot{\theta}_{5j} \\
&\vdots \quad\quad \vdots \quad\quad \vdots \\
\dddot{\psi}_{nj} &= \dddot{\theta}_{nj} - \dddot{\theta}_{5j}
\end{aligned}$$

n = link number
j = position counter
θ_{50} = initial position of link 5 in Stephenson 2

(1-1)

or

$$\begin{aligned}
\theta_{nj} &= \psi_{nj} - \psi_{3j} + \psi_{30} \\
\dot{\theta}_{nj} &= \dot{\psi}_{nj} - \dot{\psi}_{3j} \\
&\vdots \quad\quad \vdots \quad\quad \vdots \\
\dddot{\theta}_{nj} &= \dddot{\psi}_{nj} - \dddot{\psi}_{3j}
\end{aligned}$$

ψ_{30} = initial position of link 3 in the Stephenson 3

Note that θ_{5j} appears because link 5 is fixed in the Stephenson 3 and ψ_{3j} appears because link 3 is fixed in the Stephenson 2. Alternative choices for fixed links are 2 and 4. In this case the equations would contain θ_{2j} and ψ_{4j}. A general expression for any chain can be written

$$\begin{aligned}
{}^{(k)}\psi_{nj} &= {}^{(k)}\theta_{nj} - {}^{(k)}\theta_{rj} + {}^{(k)}\theta_{r0} \\
{}^{(k)}\theta_{nj} &= {}^{(k)}\psi_{nj} - {}^{(k)}\psi_{sj} + {}^{(k)}\psi_{s0}
\end{aligned}$$

n = link number
j = position counter
k = derivative counter
r = link which is fixed in mechanism with angles ψ_{nj}
s = link which is fixed in mechanism with angles θ_{nj}

(1-2)

When n = r or n = s, Eq. (1-2) becomes

$$\psi_{rj} = \theta_{r0} = \text{constant}$$

and

$$\theta_{sj} = \psi_{s0} = \text{constant}$$

which shows that r is the fixed link in the mechanism with angles ψ_{nj} and s is the fixed link in the mechanism with angles θ_{nj}. All angles are measured from the same initial absolute reference.

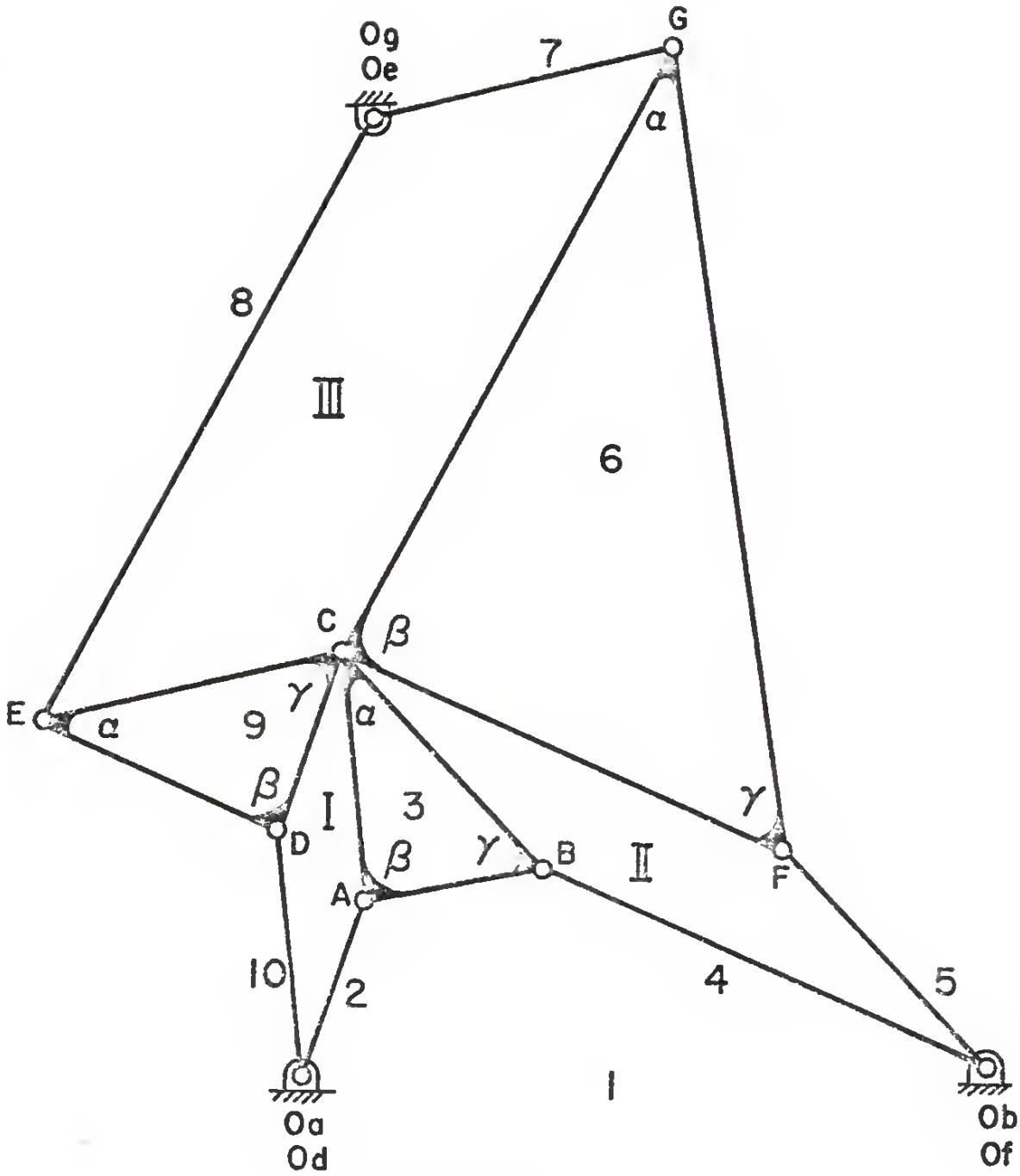
The computer program MECSYN (Chapter V) determines the inverted specifications for a general three-link chain using the above equations and an analysis procedure to determine path properties. This tool is used to synthesize a large class of multi-link mechanisms by inversion which cannot be synthesized by other closed form methods. Details of this procedure may be found in Chapters III and IV or References [14,15,19].

3. Path Cognates

Planar four-bar mechanisms exhibit a property known as the Roberts-Chebyshev theorem:

Three different planar four-bar mechanisms will trace identical coupler curves. Mechanisms related by this property are known as cognate mechanisms, Fig. (1-4). Samuel Roberts presented a proof of this theorem in 1875, while Chebyshev, using an entirely different approach, presented his proof in 1878.

Although this property is well known, some extremely important interrelations within this set of mechanisms are not widely appreciated.



{9,2,7}
{10,3,5}
{8,4,6}

Figure 1-4 Path Cognates

It is the property of the assembly that coupler links 3, 6, 9 are directly similar, that four-link loops I, II, III are parallelograms and that the angular displacement, velocity, acceleration, and all higher order properties of link sets {9,2,7}, {10,3,5} and {8,4,6} are identical.

Due to these properties of cognate mechanisms, a four-bar may be synthesized for path and angular specifications of a coupler link (3 in Fig. (1-4)) giving two additional four-bars (8,9,10,1 and 5,6,7,1) whose input links (10,5) satisfy the desired angular specifications, while the common coupler point C satisfies the desired path specifications. The result is a known coordination of a crank with the motion of a coupler path point. This tool has been extended to aid in the solution of many multi-link synthesis problems, especially the coordination of a crank with the complete motion of a body (path and angular specs) [14].

The analytical formulation for the calculation of four-bar path cognates may be found under the heading subroutine COG in Chapter V.

4. Angular Cognates

This property has been described (by Hain) as:

> The links of a four-bar mechanism may be rearranged in any order without affecting their angular motion.

Note that, in general, the same coupler point location for two angular cognate four-bars will not have the same path properties as with path cognates.

Point A on dyad (1,2), Fig. (1-5), traces a path while link 2 undergoes angular motion θ. A parallelogram (1,2,3,4) is formed by adding dyad (3,4) to dyad (1,2). Since this is a parallelogram, link 4 always remains parallel to link 2 for all motion of point A and the angular motion ϕ of link 4 is related to link 2 by a constant phase angle.

$$\phi = \theta - \delta + 180°$$

$$\dot{\phi} = \dot{\theta}$$

$$\vdots \quad \vdots$$

$$\phi^{(k)} = \theta^{(k)} \qquad \text{for all } k.$$

Since links 2,4 and links 1,3 are equal in length, this is equivalent to interchanging links 1 and 2. Applying this to four-bar (1,2,3,4) in Fig. (1-4), first to dyad BAO_a and then to dyad ABO_b demonstrates the property of angular cognates for a four-bar. However, this property exists for a single dyad alone as shown above. The importance of this property is that any mechanisms composed of dyad and four-bar chains can be synthesized for angular specifications of coupler planes and have those angular properties transformed to an input or output link using angular or path cognates, respectively. This capability is essential for the synthesis of mechanisms given dynamic specifications as will be shown in Chapter II.

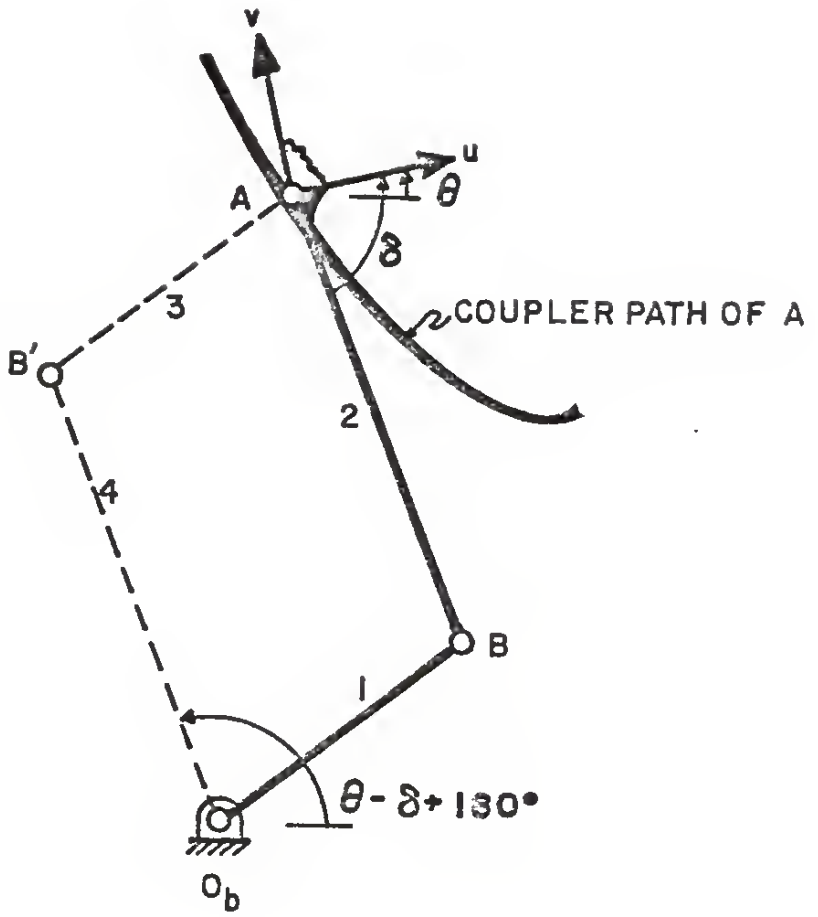


Figure 1-5 Angular Cognates

CHAPTER II

THEORY OF DYNAMIC SPECIFICATION SYNTHESIS

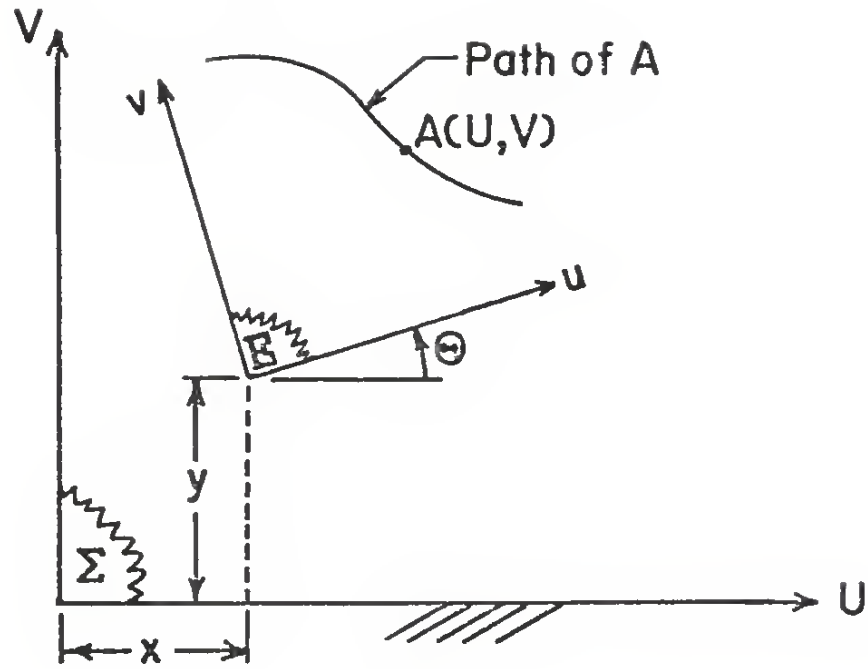
A. Introduction

The coordination of geometric motion states has been of interest to researchers in mechanisms ever since the time of Releaux. In most cases the motion is considered to be dependent on a single position parameter; i.e., the rotation angle θ for an input crank. The output state may be considered as being functionally related $\phi = f(\theta)$ to the input parameter by means of higher order properties $\phi', \phi'', \ldots$ of this function for a given value of θ. This type of motion is widely considered as the specification of infinitesimally separated positions (ISP) for the geometric motion. If, however, both input (θ) and output (ϕ) motions take on finite increments $\Delta\theta$ and $\Delta\phi$, then the specification is considered as giving finitely separated positions (FSP) for a sequence of positions $\ell = 0,1,2,\ldots$. Note that for FSP, neither parameter θ,ϕ need be considered as independent to specify the motion.

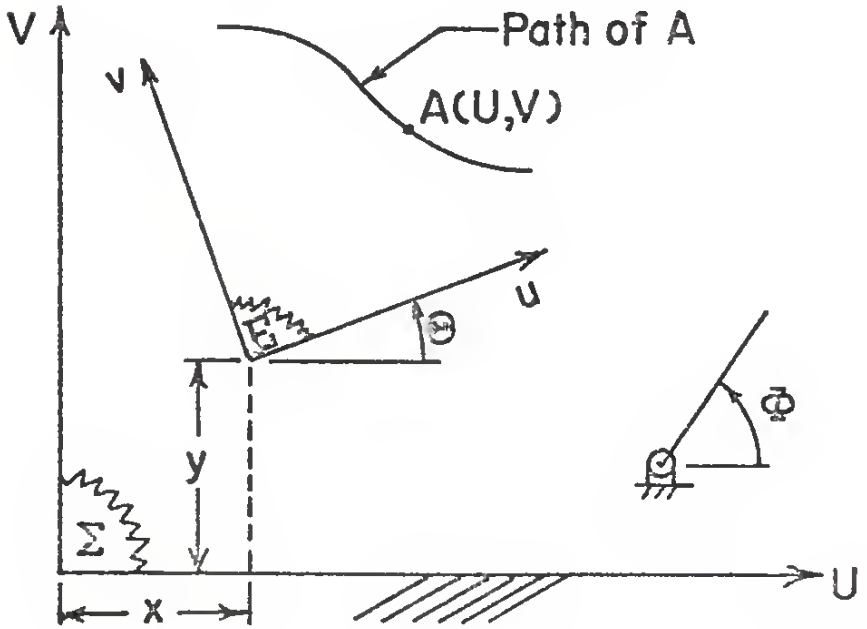
To illustrate a motion involving two functional constraints, consider the motion (U,V) of the point A(u,v) in the moving plane E where θ is the independent parameter (see Fig. 2-1a). Given the functional dependence $x = f(\theta)$, $y = f(\theta)$, due to the mechanism, it is obvious that

$$U = u \cos \theta - v \sin \theta + x(\theta)$$

$$V = u \sin \theta + v \cos \theta + y(\theta) \qquad (2\text{-}1)$$



(a)



(b)

Figure 2-1 Fixed and Moving Coordinate Systems

which is the explicit coordination of the motion of point A with the parameter θ. For ISP, one must have knowledge of the higher order properties $x', x'', \ldots y', y'' \ldots$ with respect to the independent parameter θ to obtain $U', U'', \ldots V', V'', \ldots$ by differentiating Eq. (2-1) with respect to θ. The derivatives of $x(\theta), y(\theta)$ are the constraint conditions which must be provided by the mechanism.

Now suppose the motion U,V of point A is to be coordinated with a crank parameter ϕ. This motion involves three functional constraints $x = f(\phi)$, $y = f(\phi)$ and $\theta = f(\phi)$. For ISP, it is necessary that the motion be specified in terms of higher order properties of these functions with respect to ϕ. For the system being driven by the crank parameter ϕ, an important dynamic coordination problem evolves when ϕ experiences a general input $\phi = f(t)$ with a given set of higher order properties $\dot{\phi}, \ddot{\phi}, \ldots$.

Dynamic state of a system is usually defined as the smallest collection of numbers which must be specified at time $t = t_o$ in order to be able to predict uniquely the behavior of the system for any time $t \geq t_o$ for any input belonging to the given input set provided that every element of the input set is known for $t \geq t_o$.

In the synthesis method considered here, in some cases of the motion specification, more than the dynamic state is actually prescribed. Because of this, the term dynamic specification or dynamic specs is used to indicate the prescribed set of conditions for synthesis.

For example, if the inertia force and inertia torque are prescribed for an object at a given position having mass m and rotational moment

of inertia I (the object is moving in coplanar motion), the linear and angular velocities and acceleration of the mass center may be computed. (Note that the object need not be undergoing coplanar motion but may have general spatial motion. The dynamic specs synthesis is still valid but must then be modified to use the constraint function and subsequent solution algorithms to reflect the type of spatial constraint pairs used.) Higher order properties which influence motion constrained systems may also be prescribed, such as jerk (time derivative of acceleration) and jounce (time derivative of jerk). Synthesizing a mechanism which satisfies these specifications while driven by a prescribed input determines the dynamics of the mechanism at that position. The prescribed inertia force and torque are achieved.

The solution of this problem means that the motion of the moving plane E(u,v) as $x = f(t)$, $y = f(t)$, $\theta = f(t)$ can be coordinated with a general input. The objective here is to give a fundamental procedure for treating the synthesis of analytically coupled systems for the 4-, 5-, and 6-link planar mechanisms.

B. Previous Synthesis Philosophy

If the motion depicted in Figure (2-1a) is considered as constrained, then θ may be considered as the independent motion parameter and $x = f(\theta)$, $y = f(\theta)$ become the conceptual representation of the physical constraint. The fundamental form of the physical constraint is the crank where the pin joint A(u,v) in E moves on a circular arc in the reference plane Σ. It is possible to consider a general functional constraint $F(U,V) = 0$ for the motion of point A which has the internal coefficients Q_i. Using the transform given in Eq. (2-1), the conceptual constraint becomes

$$F = f(u, v, x, y, \theta, Q_0, Q_1, Q_2, \ldots) = 0 \ . \qquad (2\text{-}2)$$

For FSP motion specifications $\{x_j, y_j, \theta_j\}$ for the motion of plane E, the problem becomes defined by

$$F_j = f(u, v, x_j, y_j, \theta_j, Q_0, Q_1, Q_2, \ldots) = 0 \qquad (2\text{-}3)$$

$$j = 0,1,2,\ldots$$

where the unknowns are the coordinates (u,v) and coefficients Q_i.
For ISP, the geometrical conditions on the motion are

$$F_k = \frac{d^k}{d\theta^k} \{f(u, v, x, y, \theta, Q_0, Q_1, Q_2, \ldots)\} = 0 \qquad (2\text{-}4)$$

$$k = 0,1,2,\ldots$$

for a given value of θ where the unknowns are the same as for FSP.

The concept of MSP implies the mixing of these conditions to provide a means of more generally specifying the motion. If the symbol PP denotes 2 ISP, and P-P denotes 2 FSP, then the following combinations

PPPPP, PPPP-P, PPP-PP, PPP-P-P

PP-PP-P, PP-P-P-P, P-P-P-P-P

represent the potential for 7 distinct motion problems. The goal should be to establish a uniform body of analytics which treats all of these problems simultaneously; i.e., the basis for the MSP concept. If ℓ counts all MSP, j counts all FSP and k counts all sequential ISP for a distinct position j, then the case symbol is sufficient to denote the kind of motion desired. For example, for case PPP-PP, the following counters apply

ℓ	0	1	2	3	4
j	0	0	0	1	1
k	0	1	2	0	1 .

Generally, the counter ℓ is understood to represent the other counters whenever the case symbol is given.

C. Statement of Dynamic Synthesis

The majority of previous synthesis problems have been referred to as kinematic synthesis solutions when in fact they have not included specifications for kinematics in the strictest definition of the term (such as velocity, acceleration, jerk, and jounce). The objective has been to synthesize mechanisms to satisfy purely geometric kinematic properties (such as the tangent, curvature, inflexion, etc., of a given function). Here the goal is to synthesize mechanisms for prescribed time dependent properties in the form of velocity, acceleration, etc., which along with specified mass and rotational inertia make up the

inertial dynamics of the coupler link. The problem will be formulated in terms of six-link mechanisms, Fig. (2-2), for two general types of solvable problems. Those in Fig. (2-3) involve the central concept of path generation with auxiliary angular motions being coordinated. Those in Fig. (2-4) deal completely with the coordination of angular motions. These solvable problems have been treated in complete detail in Ref. [15]. The level of the motion specification depends on the problem but in general involves partial dynamic specification (the dynamics related to a point or the angular dynamics) of several links in addition to the input. The prescribed quantities for these two types of problems are:

Type I (See Fig. (2-3))

$$\left.\begin{matrix} x_j, \dot{x}_j, \ldots x_j^{(k)} \\ y_j, \dot{y}_j, \ldots y_j^{(k)} \\ \theta_j, \dot{\theta}_j, \ldots \theta_j^{(k)} \end{matrix}\right\} \quad \text{Prescribed Dynamic Specifications (Add I,M) of a Moving Body}$$

$$\left.\begin{matrix} \phi_j, \dot{\phi}_j, \ldots \phi_j^{(k)} \end{matrix}\right\} \quad \text{Prescribed Coordinated Input} \qquad (2\text{-}5)$$

Type II (See Fig. (2-4))

$$\left.\begin{matrix} \theta_j, \dot{\theta}_j, \ldots \theta_j^{(k)} \\ \psi_j, \dot{\psi}_j, \ldots \psi_j^{(k)} \end{matrix}\right\} \quad \text{Prescribed Angular Dynamic Specifications for Links 1,2 (Add } I_1, I_2\text{)}$$

$$\left.\begin{matrix} \phi_j, \dot{\phi}_j, \ldots \phi_j^{(k)} \end{matrix}\right\} \quad \text{Prescribed Coordinated Input} \qquad (2\text{-}6)$$

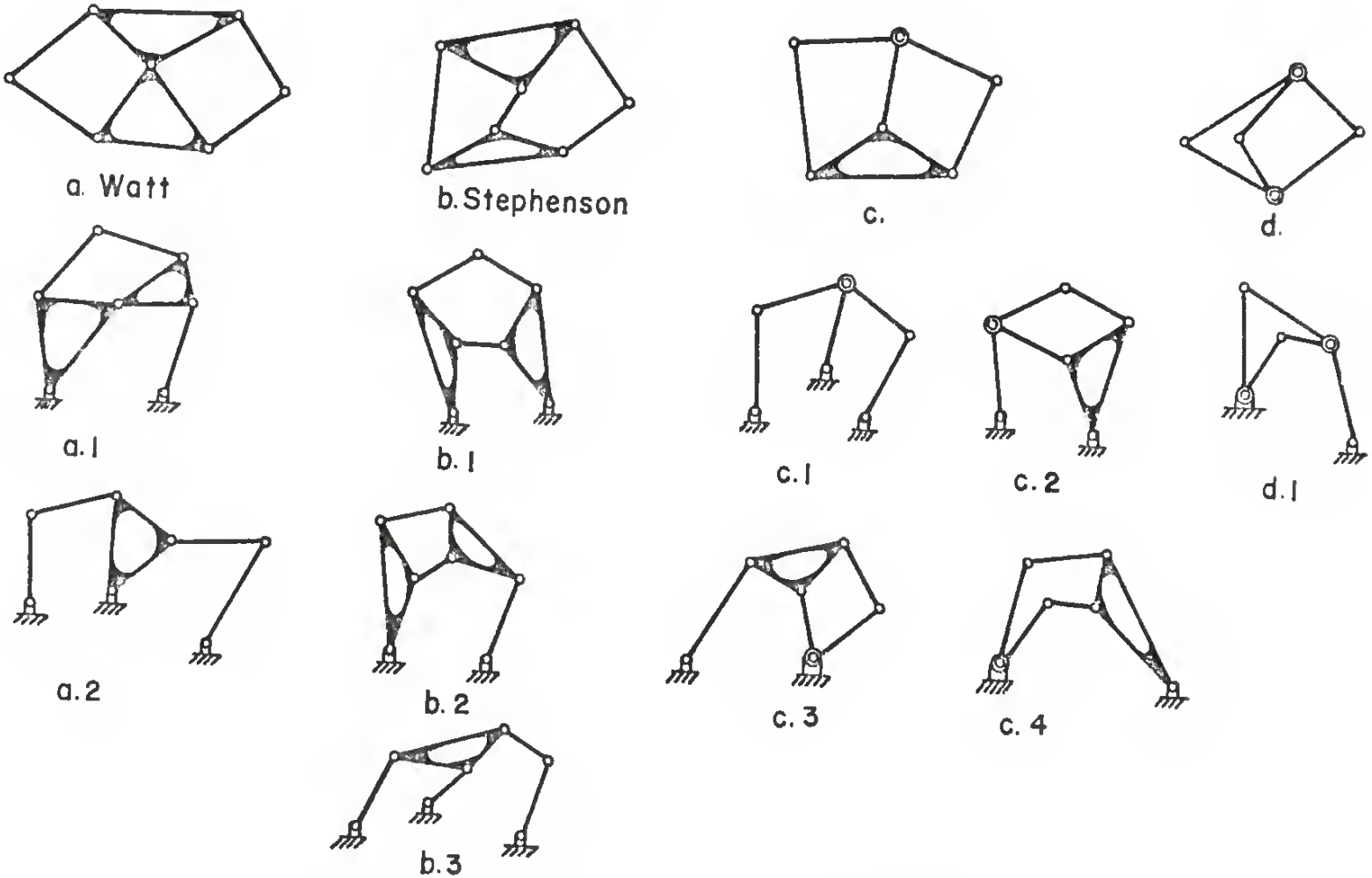
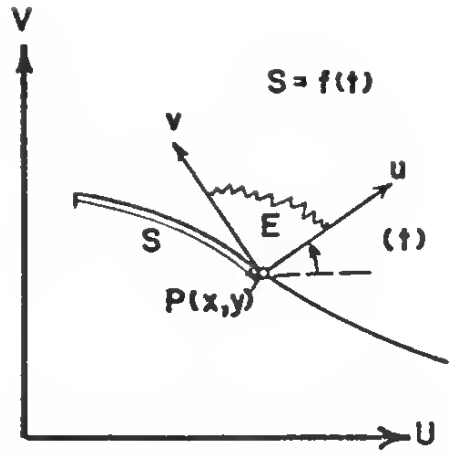
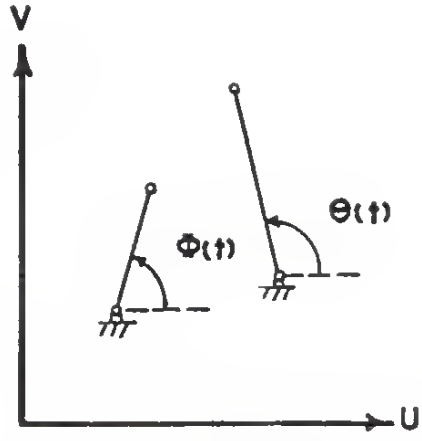


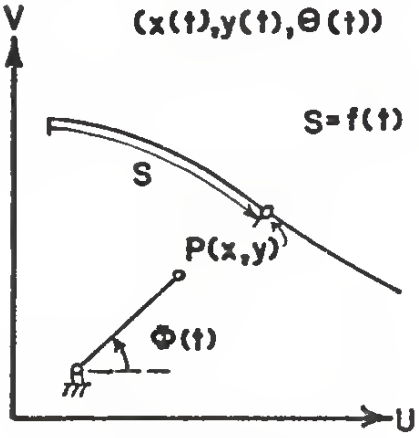
Figure 2-2 All Constrained Six-Link Mechanisms



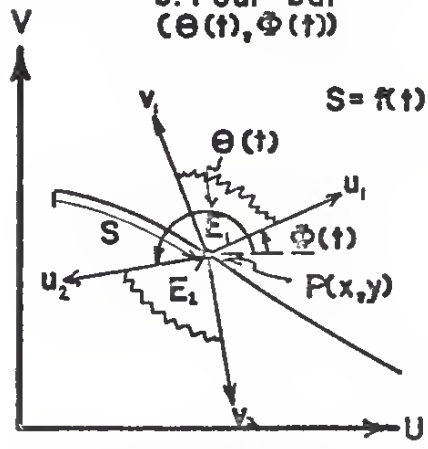
a. Four-bar
(x(t),y(t),Θ(t))



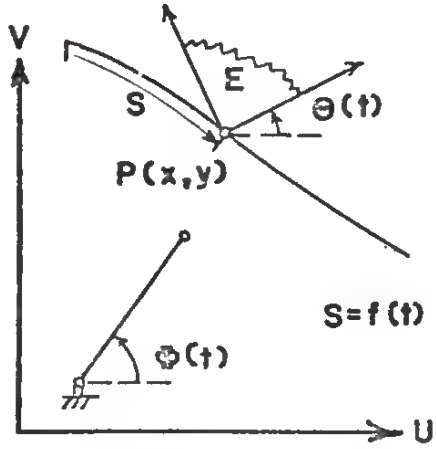
b. Four-bar
(Θ(t),Φ(t))



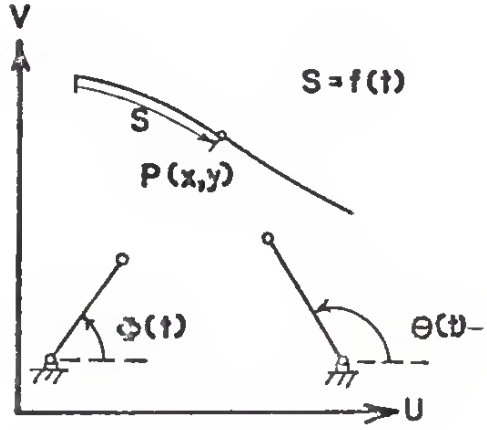
c. Four-bar
(x(t),y(t),Φ(t))



d. Stephenson I,3
(x(t),y(t), Θ(t),Φ(t))

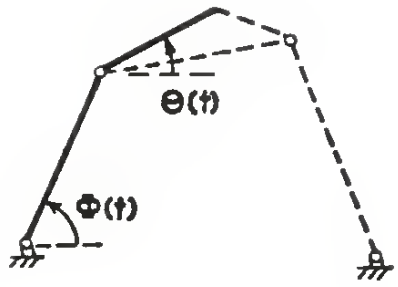


e.Stephenson I,2,3
(x(t),y(t),Θ(t),Φ(t))

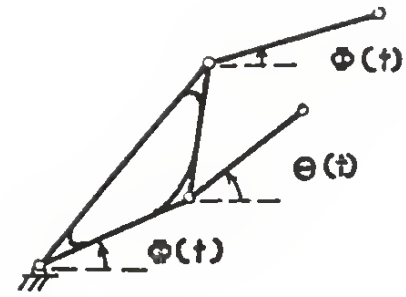


f. Stephenson I,2,3
(x(t),y(t),Θ(t),Φ(t))

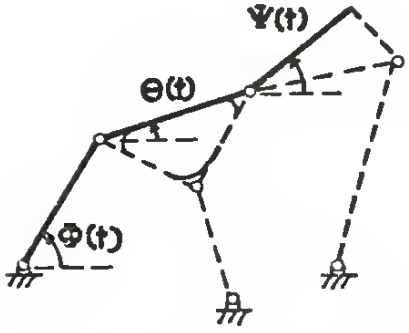
Figure 2-3 Solvable Problems - I



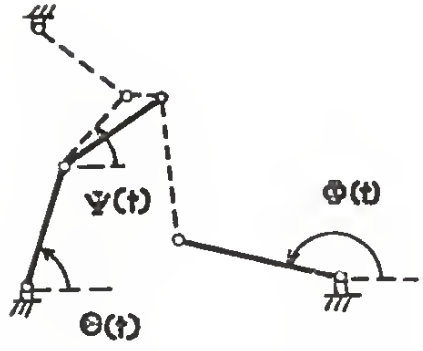
a. Four-bar
(θ(t),Φ(t))



b. Watt 1,2
(θ(t),Ψ(t),Φ(t))



c. Stephenson 1,2,3, Watt 1
(θ(t),Ψ(t),Φ(t))



d. Stephenson 1,2,3,Watt 1,2
(θ(t),Ψ(t),Φ(t))

Figure 2-4 Solvable Problems - II

where the counters (k,j) for position ℓ depend on the combination (case) of the multiply separated positions and $\overset{(k)}{x} = \frac{d^k x}{dt^k}$.

To satisfy these specifications for a six-link mechanism, it is necessary to find two or more points in the appropriate coupler links that satisfy the following conditions:

1. The points must travel on a circular arc in the reference system $\Sigma(U,V)$.

2. The points experience a time dependent motion which assures that the dynamic specs of the link are satisfied. After synthesis, this condition is met by the coordination of the mechanism with the prescribed input.

Using the same philosophy of Section B for kinematic synthesis, the dynamic synthesis constraint function for circular motion of a point can be used to establish the conditions to be met; i.e.,

$$\begin{aligned}
&\{G(x(t),\ y(t),\ \theta(t),\ \phi(t))\}\Big|_{t=t_\ell} = 0 \\
&\frac{d}{dt}\{G(x(t),\ y(t),\ \theta(t),\ \phi(t))\}\Big|_{t=t_\ell} = 0 \\
&\quad\vdots \qquad\qquad\quad \vdots \qquad\qquad\quad \vdots \\
&\frac{d^k}{dt^k}\{G(x(t),\ y(t),\ \theta(t),\ \phi(t))\}\Big|_{t=t_\ell} = 0
\end{aligned} \tag{2-7}$$

where the symbols x, y, θ, ϕ, may correspond to those in Fig. (2-1b). Satisfaction of these conditions for mixed FSP and ISP will be called dynamic specs multiply separated positions (DMSP).

The author has shown in Refs. [14,15] that this constraint link formulation or curvature relation can be decoupled for purposes of design

by means of the path point* for which the dynamic specifications are made. This process allows a modular design of five-link, six-link, and more complex mechanisms if desired. Thus, the four tools of synthesis are applied to at most a four-link chain or frequently only a dyad. Two examples will be given here which illustrate this modular approach for the Type I and Type II dynamic specs synthesis.

Conceptually, it is possible to decouple the design procedure into two or more modules which allows the formulation of two distinct sets of constraint conditions in the form:

$$\frac{d^k}{dt^k}\{G(x(t),\ y(t),\ \theta(t))\}\Big|_{t=t_\ell} = 0 \qquad (2\text{-}8)$$

and

$$\frac{d^k}{dt^k}\{G(x(t),\ y(t),\ \phi(t))\}\Big|_{t=t_\ell} = 0\ . \qquad (2\text{-}9)$$

Example 1 is associated with the Stephenson 3 for the solvable Type I problems depicted in Fig. (2-3). (See Refs. [14,15,19].) The problem specification for the problem shown in Fig. (2-5) can be established by the following steps:

Step (1) Choose the dynamic specs for the coupler link by the specs $\{x_j^{(k)}(t),\ y_j^{(k)}(t)\}$ at point C and $\{\theta_j^{(k)}(t)\}$ for the angular motion of that link. Find one constraint link (link 1) by using the dynamic curvature transform respresented by Eq. (2-8). (See Fig. (2-5a).)

---

*The path point could be a pin joint (such as C) or some other associated point in the system.

Step (2) Use the same specs for point C along with the desired input set $\phi_j^{(k)}(t)$ for a coupler link. Find two constraints (links 2 and 3) by the curvature transform related to Eq. (2-9) to form a four-bar linkage. Taking the path cognates of this four-bar results in a four-bar with cranks 2' and 3', one of which contains the set $\phi_j^{(k)}(t)$ as the input. (See Fig. (2-5b).)

Step (3) Steps (1) and (2) have created systems which have common specs at point C $\{ x_j^{(k)}(t),\ y_j^{(k)}(t)\}$. Join these at point C to obtain the desired Stephenson 3 six-link mechanism (Fig. (2-5c).)

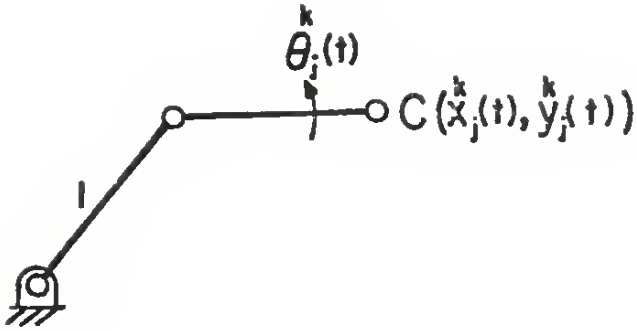
Example 2 uses the Stephenson 3 to illustrate the type of synthesis process involved with the Type II problem (see Ref. [15]) and depicted in Fig. (2-4). It involves the following steps:

Step (1) Choose the angular dynamic specs $\{ \theta_j^{(k)}(t),\ \psi_j^{(k)}(t)\}$ for two adjacent links of chosen length ℓ_1, ℓ_2 and location $A(U_a,V_a)$ in the reference system $\Sigma(U,V)$ (see Fig. (2-6a)). Analyze this dyad to determine the dynamic specs of point C as

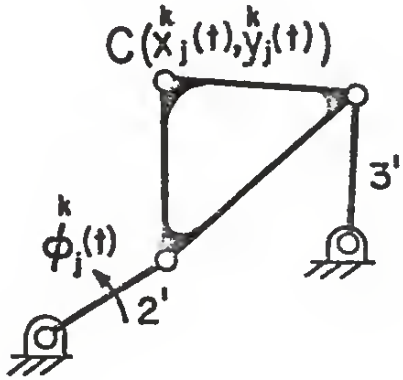
$$x_j^{(k)}(t) = \frac{d^k}{dt^k}\{\ell_1 \cos\theta + \ell_2 \cos\psi + U_a\}\Big|_{t=t_j}$$

$$y_j^{(k)}(t) = \frac{d^k}{dt^k}\{\ell_1 \sin\theta + \ell_2 \sin\psi + V_a\}\Big|_{t=t_j} .$$

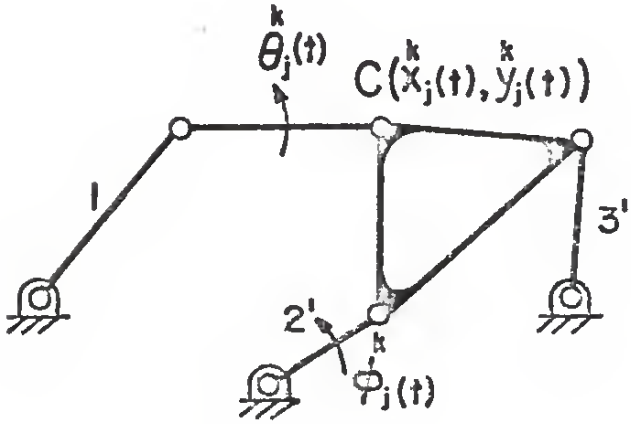
Step (2) Specify the input set $\phi_j^{(k)}(t)$ desired to drive the system along with the specs of point C to create the dynamic specs



a.

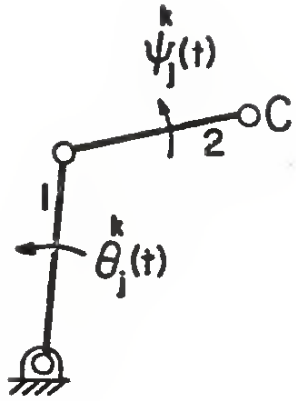


b.

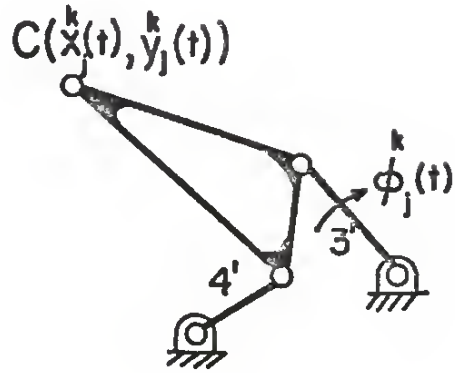


c.

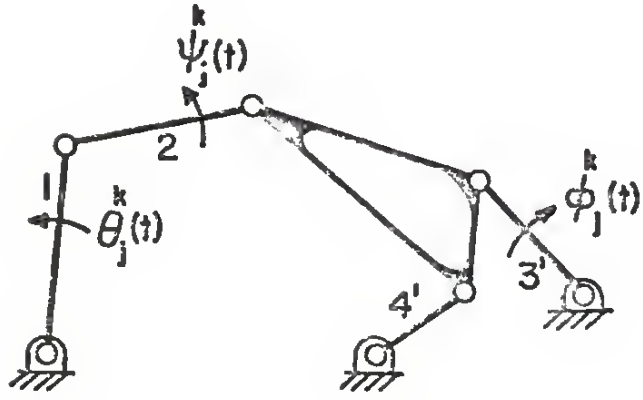
Figure 2-5 Type I Synthesis Problem



a.



b.



c.

Type 2-6 Type II Synthesis Problem

for a coupler link. Using the curvature transform represented by Eq. (2-9) for the specs $\{ x_j^{(k)}(t),\ y_j^{(k)}(t),\ \phi_j^{(k)}(t)\}$, find two constraint links to form a four-bar linkage. Taking the path cognate of this linkage results in a four-bar with cranks 3' and 4', one of which contains the set $\phi_j^{(k)}(t)$ as shown in Fig. (2-6b).

Step (3) Steps 1 and 2 contain point C with common specs $\{ x_j^{(k)}(t),\ y_j^{(k)}(t)\}$. Joining these systems as shown in Fig. (2-6c) gives the desired Stephenson 3 mechanism.

In this fashion more than 35 distinct six-link synthesis problems according to the step-by-step procedures of Refs. [14,15,19] are decoupled into component problems of low complexity. The examples here illustrate the least difficult of these procedures. Chapter IV will consider more involved procedures to completely validate the generality of this section. This generality is distinguished from the author's previous work by the dynamic curvature transform which will be presented in the next section. All the other tools of synthesis (inversion, angular cognates, and path cognates) maintain their previous significance.

D. Formulation of the Dynamic Curvature Transform

The constraint conditions used to synthesize mechanisms depends on a circular constraint which can be represented as

$$g(U,V) = Q_0(U^2+V^2) + 2Q_1U + 2Q_2V + Q_3 = 0 \qquad (2\text{-}10)$$

Table 2-1 SYNTHESIS COEFFICIENTS

k	$D_{0\ell}$	$D_{1\ell}$	$D_{2\ell}$	$D_{3\ell}$	$D_{4\ell}$	$D_{5\ell}$	$D_{6\ell}$
0	$\frac{1}{2}[(x_j^2+y_j^2)-(x_0^2+y_0^2)]$	$(x_j\cos\theta_j+y_j\sin\theta_j)$ $-(x_0\cos\theta_0+y_0\sin\theta_0)$	$(y_j\cos\theta_j-x_j\sin\theta_j)$ $-(y_0\cos\theta_0-x_0\sin\theta_0)$	$\cos\theta_j-\cos\theta_0$	$\sin\theta_j-\sin\theta_0$	x_j-x_0	y_j-y_0
1	$x\dot{x}+y\dot{y}$	$(-x\dot{\theta}+\dot{y})\sin\theta$ $+(\dot{x}+y\dot{\theta})\cos\theta$	$(-\dot{x}-y\dot{\theta})\sin\theta$ $+(-x\dot{\theta}+\dot{y})\cos\theta$	$-\dot{\theta}\sin\theta$	$\dot{\theta}\cos\theta$	$\dot{x}$	$\dot{y}$
2	$\dot{x}^2+x\ddot{x}+\dot{y}^2+y\ddot{y}$	$(-x\ddot{\theta}-2\dot{x}\dot{\theta}-y\dot{\theta}^2+\ddot{y})\sin\theta$ $+(-x\dot{\theta}^2+\ddot{x}+y\ddot{\theta}+2\dot{y}\dot{\theta})\cos\theta$	$(x\dot{\theta}^2-\ddot{x}-y\ddot{\theta}-2\dot{y}\dot{\theta})\sin\theta$ $+(-x\ddot{\theta}-2\dot{x}\dot{\theta}-y\dot{\theta}^2+\ddot{y})\cos\theta$	$-\ddot{\theta}\sin\theta$ $-\dot{\theta}^2\cos\theta$	$-\dot{\theta}^2\sin\theta$ $+\ddot{\theta}\cos\theta$	$\ddot{x}$	$\ddot{y}$
3	$3\dot{x}\ddot{x}+x\dddot{x}+3\dot{y}\ddot{y}+y\dddot{y}$	$(x\dot{\theta}^3-x\dddot{\theta}-3\dot{x}\ddot{\theta}-3\ddot{x}\dot{\theta}$ $-3y\dot{\theta}\ddot{\theta}-3\dot{y}\dot{\theta}^2+\dddot{y})\sin\theta$ $+(-3x\dot{\theta}\ddot{\theta}-3\dot{x}\dot{\theta}^2+\dddot{x}-y\dot{\theta}^3$ $+y\dddot{\theta}+3\dot{y}\ddot{\theta}+3\ddot{y}\dot{\theta})\cos\theta$	$(3x\dot{\theta}\ddot{\theta}+3\dot{x}\dot{\theta}^2-\dddot{x}+y\dot{\theta}^3$ $-y\dddot{\theta}-3\dot{y}\ddot{\theta}-3\ddot{y}\dot{\theta})\sin\theta$ $+(x\dot{\theta}^3-x\dddot{\theta}-3\dot{x}\ddot{\theta}-3\ddot{x}\dot{\theta}$ $-3y\dot{\theta}\ddot{\theta}-3\dot{y}\dot{\theta}^2+\dddot{y})\cos\theta$	$(\dot{\theta}^3-\dddot{\theta})\sin\theta$ $+(-3\dot{\theta}\ddot{\theta})\cos\theta$	$(-3\dot{\theta}\ddot{\theta})\sin\theta$ $+(-\dot{\theta}^3+\dddot{\theta})\cos\theta$	$\dddot{x}$	$\dddot{y}$
4	$3\ddot{x}^2+4\dot{x}\dddot{x}+x\ddddot{x}$ $+3\ddot{y}^2+4\dot{y}\dddot{y}+y\ddddot{y}$	$(-x\ddddot{\theta}+6x\dot{\theta}^2\ddot{\theta}+4x\dot{\theta}^3$ $-4\dot{x}\dddot{\theta}-6\ddot{x}\ddot{\theta}-4\dddot{x}\dot{\theta}+y\dot{\theta}^4$ $-4y\dot{\theta}\dddot{\theta}-3y\ddot{\theta}^2-12\dot{y}\dot{\theta}\ddot{\theta}$ $-6\ddot{y}\dot{\theta}^2+\dddot{y})\sin\theta$ $+(x\dot{\theta}^4-4x\dot{\theta}\dddot{\theta}-3x\ddot{\theta}^2$ $-12\dot{x}\dot{\theta}\ddot{\theta}-6\ddot{x}\dot{\theta}^2+\dddot{x}$ $+y\dddot{\theta}-6y\dot{\theta}^2\ddot{\theta}-4y\dot{\theta}^3$ $+4\dot{y}\dddot{\theta}+6\ddot{y}\ddot{\theta}$ $+4\dddot{y}\dot{\theta})\cos\theta$	$(-x\dot{\theta}^4+4x\dot{\theta}\dddot{\theta}+3x\ddot{\theta}^2$ $+12\dot{x}\dot{\theta}\ddot{\theta}+6\dot{x}\dot{\theta}^2-\ddddot{x}$ $-y\dddot{\theta}+6y\dot{\theta}^2\ddot{\theta}+4\dot{y}\dot{\theta}^3$ $-4\dot{y}\dot{\theta}^3-6\ddot{y}\dot{\theta}-4\dddot{y}\dot{\theta})\sin\theta$ $+(-x\dddot{\theta}+6x\dot{\theta}^2\ddot{\theta}+4\dot{x}\dot{\theta}^3$ $-4\dot{x}\dddot{\theta}-6\ddot{x}\ddot{\theta}-4\dddot{x}\dot{\theta}$ $+y\dot{\theta}^4-4y\dot{\theta}\dddot{\theta}-3y\ddot{\theta}^2$ $-12\dot{y}\dot{\theta}\ddot{\theta}-6\ddot{y}\dot{\theta}^2$ $+\dddot{y})\cos\theta$	$(6\dot{\theta}^2\ddot{\theta}-\dddot{\theta})\sin\theta$ $+(-4\dot{\theta}\dddot{\theta}-3\ddot{\theta}^2$ $+\dot{\theta}^4)\cos\theta$	$(-3\ddot{\theta}^2-4\dot{\theta}\dddot{\theta}$ $+\dot{\theta}^4)\sin\theta$ $+(-6\dot{\theta}^2\ddot{\theta}+\dddot{\theta})\cos\theta$	$\ddddot{x}$	$\ddddot{y}$

$\dot{x}=\frac{dx}{dt},\ldots,\overset{(k)}{x}=\frac{d^kx}{dt^k}$. All $\overset{(k)}{x}$, $\overset{(k)}{y}$, $\overset{(k)}{\theta}$ after k = 0 are evaluated at the j^{th} finita position.

which for dynamic constraint conditions requires that

$$g_\ell = \frac{d^k}{dt^k}\{Q_0(U^2+V^2) + 2Q_1U + 2Q_2V + Q_3\}\Big|_{t=t_\ell} = 0 \qquad (2\text{-}11)$$

$$\text{for} \quad \ell = 0,1,2,\ldots .$$

Subtracting the condition for $\ell = 0$ gives the generalized circle constraint function

$$G_\ell = g_\ell - g_0 = \frac{d^k}{dt^k}\{Q_0(U^2+V^2) + 2Q_1U + 2Q_2V\}\Big|_{t=t_\ell} = 0 \qquad (2\text{-}12)$$

$$\text{for} \quad \ell = 1,2,3,\ldots$$

which removes the coefficient Q_3 from further consideration. Using the coordinate transformation $(U,V) \rightarrow (u,v)$ given in the introduction transforms this system into the form

$$G_\ell = D_{0\ell}Q_0 + D_{1\ell}uQ_0 + D_{2\ell}vQ_0 + D_{3\ell}(uQ_1+vQ_2)$$
$$+ D_{4\ell}(uQ_2-vQ_1) + D_{5\ell}Q_1 + D_{6\ell}Q_2 = 0 \qquad (2\text{-}13)$$
$$\ell = 1,2,3,4$$

where ℓ is the counter corresponding to the (k,j) indices for the case of DMSP. The fundamental coefficients $D_{m\ell}$ may be considered as the dynamic circular constraint coefficients and are listed for all possible (k,j) values in Table 2-1. To establish the constraint condition G_ℓ, one chooses the $D_{m\ell}$ from the table according to the indices (k,j) for the value of the counter ℓ. These coefficients are completely general and are applicable for any location of the reference systems E(u,v) and Σ(U,V). All the $D_{m\ell}$ have been tabulated in a single computer program. Since all cases of DMSP are represented simultaneously by the $D_{m\ell}$, a considerable savings in programming complexity results. In addition, all synthesis equations can be formulated explicitly in terms of the

$D_{m\ell}$ making it possible to create a single body of analytics to cover all cases. This means that one (not seven) computer program is sufficient to treat DMSP. The modular synthesis approach based on sub-unit link groups (the dyad and the four-bar) allows synthesis of many different types of planar mechanisms in terms of a relatively small computer program rather than the cumbersome alternative of a program for each mechanism.

Equation (2-13) consists of four equations which are nonlinear in the circle point coordinates (u,v) and the centerpoint coordinates ($U = -\frac{Q_1}{Q_0}$, $V = -\frac{Q_2}{Q_0}$) of the circle on which the circle point moves (see Eq. (2-10)). To obtain a solution, a mapping into a new set of variables Z_m (m = 0,1,...6) is defined by

$$\begin{aligned} Z_0 &= Q_0 & Z_3 &= uQ_1 + vQ_2 & Z_5 &= Q_1 \\ Z_1 &= uQ_0 & Z_4 &= uQ_2 - vQ_1 & Z_6 &= Q_2 \\ Z_2 &= vQ_0 \end{aligned} \qquad (2\text{-}14)$$

which means that Eq. (2-13) can be written as a linear system of equations in the form

$$\sum_{m=0}^{6} D_{m\ell} Z_m = 0 \qquad \ell = 1,2,3,4 \; . \qquad (2\text{-}15)$$

Two additional unknowns have now been introduced to allow the formulations

$$\begin{aligned} Z_0Z_3 &= Z_1Z_5 + Z_2Z_6 \\ Z_0Z_4 &= Z_1Z_6 - Z_2Z_5 \; . \end{aligned} \qquad (2\text{-}16)$$

Equations (2-15) and (2-16) constitute six equations in the six variables Z_m/Z_0, m = 1,2,...6 yielding four sets of desired solutions

$$\left.\begin{aligned} u_n = \left(\frac{Z_1}{Z_0}\right)_n, \quad v_n = \left(\frac{Z_2}{Z_0}\right)_n &\Big\} \text{Circle Points} \\ -\left(\frac{Q_1}{Q_0}\right)_n = -\left(\frac{Z_5}{Z_0}\right)_n, \quad -\left(\frac{Q_1}{Q_0}\right)_n = -\left(\frac{Z_6}{Z_0}\right) &\Big\} \text{Center Points} \end{aligned}\right\} \begin{matrix} \text{Dynamic} \\ \text{Specifications} \\ \text{Constraint} \\ \text{Links} \end{matrix} \tag{2-17}$$

These pairs of points could be considered to be generalized Burmester pairs for dynamic synthesis.

The actual solution of Eqs. (2-15) and (2-16) is established by first solving the linear system of Eq. (2-15) for the unknowns Z_3, Z_4, Z_5, Z_6 in terms of the unknowns Z_0, Z_1, Z_2 in the form

$$\begin{bmatrix} D_{31} & D_{41} & D_{51} & D_{61} \\ D_{32} & D_{42} & D_{52} & D_{62} \\ D_{33} & D_{43} & D_{53} & D_{63} \\ D_{34} & D_{44} & D_{54} & D_{64} \end{bmatrix} \begin{bmatrix} Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} = - \begin{bmatrix} D_{01} & D_{11} & D_{21} \\ D_{02} & D_{12} & D_{22} \\ D_{03} & D_{13} & D_{23} \\ D_{04} & D_{14} & D_{24} \end{bmatrix} \begin{bmatrix} Z_0 \\ Z_1 \\ Z_2 \end{bmatrix} \tag{2-18}$$

or

$$\mathbb{D}\,\underline{Z} = \underline{b} \tag{2-19}$$

where the j,k correspond to the ℓ counter for the case being considered. When $\mathbb{D}$ is non-singular, the solution is

$$Z_m = Z_0 A_m + Z_1 B_m + Z_2 C_m \qquad m = 3,4,5,6 \tag{2-20}$$

and

$$A_m = -\frac{1}{|\mathbb{D}|} \sum_{\ell=1}^{4} D_{0\ell}\, \Delta_{m\ell}, \qquad B_m = -\frac{1}{|\mathbb{D}|} \sum_{\ell=1}^{4} D_{1\ell}\, \Delta_{m\ell},$$

$$C_m = \frac{1}{|\mathbb{D}|} \sum_{\ell=1}^{4} D_{2\ell}\, \Delta_{m\ell}.$$

$\mathbb{D} = \|d_{m\ell}\|$, $\Delta_{m\ell}$ is the cofactor of element $d_{m\ell}$.

When $\mathbb{D}$ is singular as it is in the case of a specified motion for a point along a line, then the unknowns Z_1, Z_2, Z_3, Z_4 are eliminated in favor of Z_0, Z_5, Z_6. This alternative process treats most special cases.

Eliminating Z_3, Z_4, Z_5, Z_6 between Eq. (2-16) and Eq. (2-20) gives two general conics in the coordinates $u = Z_1/Z_0$ and $v = Z_2/Z_0$:

$$\begin{aligned} av^2 + b\,uv + c\,v + d\,u^2 + e\,u + f &= 0 \\ a'v^2 + b'uv + c'v + d'u^2 + e'u + f' &= 0 \end{aligned} \tag{2-21}$$

$$a = C_4,\quad b = C_3+B_4,\quad c = A_4-C_1,\quad d = B_3,\quad e = A_3-B_1,\quad f = -A_2$$

$$a' = C_3,\quad b' = B_3-C_4,\quad c' = A_3-C_2,\quad d' = -B_4,\quad e' = B_2-A_4,\quad f' = A_2.$$

Using the Bezout Eliminant [23] for Eq. (2-21), a quartic polynomial in u results as

$$E_4u^4 + E_3u^3 + E_2u^2 + E_1u + E_0 = 0\ . \tag{2-22}$$

If $(ab') = ab' - a'b$, the coefficients may be written in the form

$$\begin{aligned} E_4 &= (ad')^2 - (ab')(bd') \\ E_3 &= 2(ad')(ae') - (ab')[(be') + (cd')] - (ac')(bd') \\ E_2 &= (ae')^2 + a(ad')(af') - (ab')[(bf') + (ce')] \\ &\qquad - (ac')[(be') + (cd')] \\ E_1 &= 2(ae')(af') - (ac')[(bf') + (ce')] - (ab')(cf') \\ E_0 &= (af')^2 - (ac')(cf'). \end{aligned} \tag{2-23}$$

The common root v is given by

$$v = \frac{-[(ad')u^2 + (ae')u + (af')]}{[(ab')u + (ac')]}\ . \tag{2-24}$$

The Burmester circle points for the dynamic specs problem are (u_n, v_n) corresponding to the max (n) = 0, 2, or 4 solutions of Eq. (2-22). The associated center points to complete the data in Σ for the constraint pair are

$$U_n = -(A_5 + B_5 u_n + C_5 v_n) \tag{2-25}$$

$$V_n = (A_6 + B_6 u_n + C_6 v_n) \qquad \max(n) = 0,2,4 \ .$$

For motion of a point along a line, Eq. (2-22) reduces to a cubic and max (n) = 1 or 3.

E. Problem Concepts

Ordinary geometric or kinematic synthesis may be considered as a special case of this formulation and can easily be obtained by replacing t by θ ($t \sim \theta$) and by requiring that

$$t \sim \theta, \quad \frac{d\theta}{dt} \sim \frac{d\theta}{d\theta} = 1, \quad \frac{d^k\theta}{dt^k} \sim \frac{d^k\theta}{d\theta^k} = 0 \qquad k > 1 \tag{2-26}$$

such that θ becomes the independent parameter in the constraint function

$$G_\ell = \frac{d^k}{dt^k}\{G(x(t), y(t), \theta(t))\}\Big|_{t=t_\ell} = 0 \tag{2-27}$$

which now becomes

$$G_\ell = \frac{d^k}{d\theta^k}\{G(x(\theta), y(\theta), \theta)\}\Big|_{\theta=\theta_\ell} = 0 \ . \tag{2-28}$$

The dynamic coefficients $D_{m\ell}$ of Table 2-1 automatically reduce to the $A_{m\ell}$ of kinematic synthesis for MSP found in Ref. [9].

Another useful property of this formulation is easily obtained merely by changing the input specifications to the program. This

involves the geometric synthesis for MSP of complex six-link mechanisms. The previously mentioned Type I and Type II problems now become specified as

$$\underline{\text{Type I}} \qquad \left.\begin{aligned} & x_j(\phi),\ \left(\frac{dx}{d\phi}\right)_j,\ \ldots\left(\frac{d^k x}{d\phi^k}\right)_j \\ & y_j(\phi),\ \left(\frac{dy}{d\phi}\right)_j,\ \ldots\left(\frac{d^k y}{d\phi^k}\right)_j \\ & \theta_j(\phi),\ \left(\frac{d\theta}{d\phi}\right)_j,\ \ldots\left(\frac{d^k \theta}{d\phi^k}\right)_j \end{aligned}\right\} \quad \text{Prescribed Geometric Specs of a Moving Body}$$

$$\underline{\text{Type II}} \qquad \left.\begin{aligned} & \theta_j(\phi),\ \left(\frac{d\theta}{d\phi}\right)_j,\ \ldots\left(\frac{d^k \theta}{d\phi^k}\right)_j \\ & \psi_j(\phi),\ \left(\frac{d\psi}{d\phi}\right)_j,\ \ldots\left(\frac{d^k \psi}{d\phi^k}\right)_j \end{aligned}\right\} \quad \text{Prescribed Geometric Angular Specs}$$

where the independent parameter ϕ_j takes on increments for FSP only (i.e., $\phi_j = \phi_\ell$). The Type I problem is the coordination of a coupler path $x(\phi)$, $y(\phi)$ with a coupler angle $\theta(\phi)$ (or output angle) and an input angle ϕ. The Type II problem is the coordination of three angular quantities with ϕ the input. The kinematic problems of the Type I and Type II specifications which were not previously treated by the author [15] can now be immediately solved in the following manner.

The independent parameter t may be considered as a dummy parameter and replaced by an input angle ($t \sim \theta$) while reinterpreting Eq. (2-7) as

$$\frac{d^k}{d\theta^k}\{G(x(\theta),\ y(\theta),\ \theta,\ \phi(\theta))\}\Big|_{\theta=\theta_\ell} = 0 \quad . \tag{2-29}$$

This interpretation is used in modular form in three steps similar to the use of Eqs. (2-8) and (2-9) in Example 1 of Section C to design complex six-link mechanisms. The condition following from Eq. (2-8) for the first step becomes

$$\frac{d^k}{d\theta^k}\{G(x(\theta),\ y(\theta),\ \theta)\}\Big|_{\theta=\theta_\ell} = 0 \tag{2-30}$$

which reduces to an ordinary kinematic (or geometric) synthesis problem and can be solved in the usual manner (i.e., by using the MSP formulation in Ref. [9] or the complex vector loop equation in Ref. [24]) or by using the "dynamic MSP" synthesis program with the specifications

$$\frac{d^k x_j(t)}{dt^k} \to \frac{d^k x_j(\theta)}{d\theta^k}\ ,\quad \frac{d^k y_j(t)}{dt^k} \to \frac{d^k y_j(\theta)}{d\theta^k} \tag{2-31}$$

$$\theta,\ \frac{d\theta}{dt} \sim \frac{d\theta}{d\theta} = 1,\quad \frac{d^2\theta}{dt^2} = \frac{d^2\theta}{dt^3} = \frac{d^4\theta}{dt^4} = 0$$

in the computer program input. The second step cannot be solved by the usual kinematic synthesis methods and must be done by the "dynamic MSP" formulation. In Step 2, Eq. (2-9) becomes

$$\frac{d^k}{d\theta^k}\{G(x(\theta),\ y(\theta),\ \phi(\theta))\}\Big|_{\theta=\theta_\ell} \tag{2-32}$$

which is distinct from Eq. (2-30). Since t is replaced by θ,($t \sim \theta$), the specifications are modified to

$$\frac{d^k x_j(t)}{dt^k} \to \frac{d^k x_j(\theta)}{d\theta^k}\ ,\quad \frac{d^k y_j(t)}{dt^k} \to \frac{d^k y_j(\theta)}{d\theta^k},\quad \frac{d^k \phi_j(t)}{dt^k} \to \frac{d^k \phi_j(\theta)}{d\theta^k}$$

where the indices j,k depend on the counter ℓ and the case being considered.

Part II presents two practical applications of dynamic specs synthesis (DMSP). The first example utilizes the full capabilities of the method in designing a Stephenson 3 and a Stephenson 2 for the same dynamic specifications. The second example of geometric MSP for kinematic synthesis as described in terms of Eqs. (2-30) and (2-32) uses the dynamic specs formulation.

The philosophy of dynamic specs synthesis shows that a large range of unsolved problems can now be solved for not only coordinated geometric specs but dynamic specs as well. Because of the generality of the $D_{m\ell}$, a single body of analytics, uniform in its format is now made available in terms of a single computational tool. As stressed in Refs. [15,19] and extended in Ref. [14], the principal task of the designer is the determination of the proper set of specifications for a given mechanism constraint to satisfy the posed problem. For geometric synthesis it has been possible to establish all properties of the system (such as pole locations, generalized inflection circles, cubic circle point curves, Burmester point coordinates, etc.) in terms of the $A_{m\ell}$. Because the concept of dynamic synthesis is a logical extension of geometric synthesis, it is clear that the $D_{m\ell}$ will allow the same explicit formulation.

The author has developed a complete and easily used computational tool for the expanded concept of DMSP. This computer program is called MECSYN and will soon be made available with a complete user's manual to assist the designer in its operation. MECSYN is a non-iterative, efficient program containing the four- and five-position dynamic specs MSP curvature transform for all seven cases in addition to path cognates,

angular cognates, inversion, link length ratio and Grashof criteria calculation. All special cases are included such as straight line motion. MECSYN also contains an analysis package which allows the designer to investigate the usefulness of the synthesized mechanisms. Use of MECSYN will be described in Chapter V along with a listing of the current version.

CHAPTER III

DYNAMIC SPECS SYNTHESIS OF FOUR-, FIVE- AND SIX-LINK MECHANISMS

In this chapter, the synthesis procedures will be developed and illustrated by examples for representative mechanisms (as shown in Fig. (2-3) and Fig. (2-4) and cases utilizing the four tools of dynamic synthesis as described in Chapters I and II. First, a four-bar dynamic specs "function generator" is designed for Case 2 (PPPP-P) using both angular and path cognates. Next, a Stephenson-2 six-link mechanism is designed to coordinate the angular dynamic specs of three links for Case 1 (PPPPP). Finally, a geared five-link mechanism is designed to coordinate the input with the dynamic specs of a coupler link for Case 4 (PPP-PP). The examples in this chapter are verified by two distinct analysis methods, one numerical and one closed form and non-iterative. The synthesis procedures for the application examples in Chapter IV are verified by a third distinct non-iterative analysis method. Also, the examples in Chapter IV demonstrate the remaining significant case of dynamic specs synthesis, i.e., Case 3 (PPP-PP). The other cases are: Case 7 (P-P-P-P-P) which is purely geometric kinematic synthesis by definition, since it involves specification of finitely separated positions (FSP) only; Case 5 (PP-PP-P) and Case 6 (PP-P-P-P) which involve specification of finitely separated positions (P-P) or velocity distributions (PP). Since geometric kinematic synthesis is

a special case of dynamic specs synthesis all seven of these cases are significant for the specification of geometric kinematic properties such as position, tangent, curvature and rate of change of curvature referred to as multiply separated positions (MSP).

A. Four-Bar Dynamic Specs "Function Generator"

The name "function generator" is the classical name used for a mechanism whose output link position parameter approximates a prescribed function with respect to its input link position parameter, i.e., pure geometric kinematic synthesis. Regardless of what the input dynamics are, the mechanism still generates the same geometric function. In the context of dynamic specs synthesis, it is a misnomer. In fact, a four-bar synthesized to coordinate the input* with the dynamic specs of the output approximates two functions, output vs. time and input vs. time. This again illustrates the dependence of the dynamic specs synthesis on the appropriate time dependent input. The resulting mechanism always satisfies the same output vs. input function regardless of what time dependent input is provided. This follows since the mechanism function is only dependent on the invariant link dimensions. The name function generator is used here to draw attention to the contrast between classical geometric kinematic synthesis and dynamic specs synthesis.

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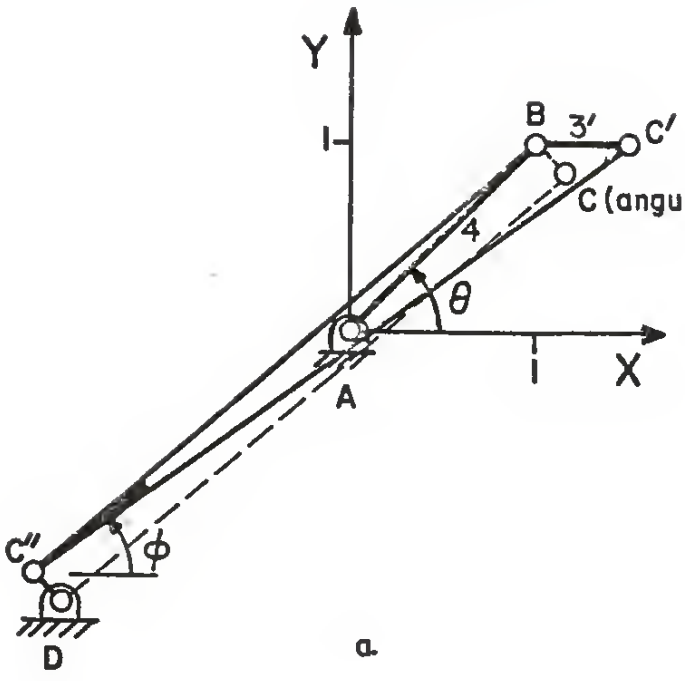
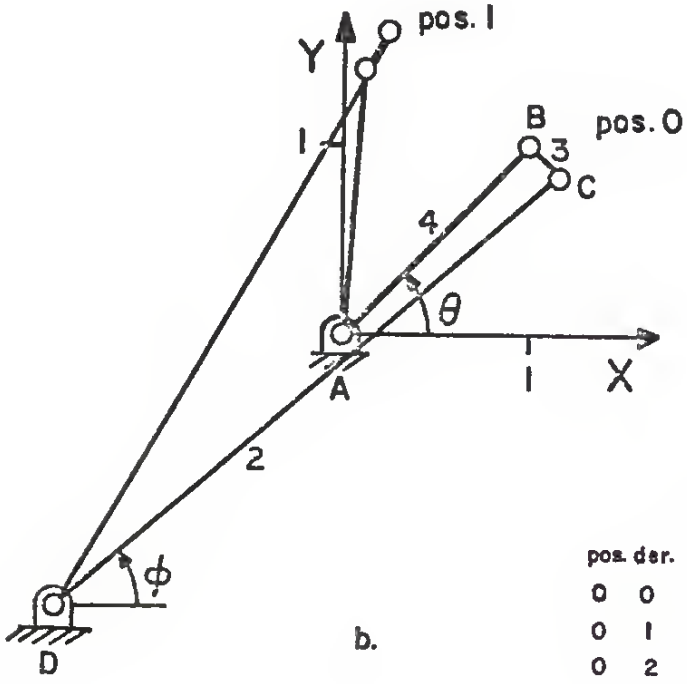
*Input and output refer to links connected to ground as opposed to coupler links--not connected directly to ground.



A = (0,0)
B = (0,1)
C' = (1.5,1)
C'' = (-1.695,-1.301)
C = (1.16,.837)
D = (-1.535,-1.464)



CASE 2

pos.	der.	θ	φ
0	0	45°	0°
0	1	5 rad/sec	2 rad/sec
0	2	10 rad/sec^2	4 rad/sec^2
0	3	15 rad/sec^3	8 rad/sec^3
1	0	85°	20°

Figure 3-1 Four-Bar Function Generator by Angular Cognates

Rather than using inversion to synthesize the four-bar function generator as has been done in the past, a new method will be used as presented by the author and Tesar (Ref. [15]) using angular cognates. The advantage of this method (which is less difficult conceptually than inversion) is that it gives an alternate set of solutions. This is not the case with the inversion method, since the angular specifications for one of the inverted links is a function of both input and output specifications.

The case treated here involves the specification of velocity, acceleration and jerk at the first position and a second position. Refer to Fig. (3-1) in the following design steps:

Step 1. Prescribe the angular dynamic specs of two adjacent links (4,3'), as $\{ \theta_j^{(k)}(t), \phi_j^{(k)}(t) \}$ (input and output) and also the location of points A, B and C' as shown in Fig. (3-1):

ℓ j k	$\theta_j^{(k)}(t)$ - input	$\phi_j^{(k)}(t)$ - output
0 0 0	45°	0°
1 0 1	5 rad/sec	2 rad/sec
2 0 2	10 rad/sec^2	4 rad/sec^2
3 0 3	15 rad/sec^3	8 rad/sec^3
4 1 0	85°	20°

A = (0, 0), B = (0, 1), C' = (1.5, 1).

Note that the prescribed initial angle θ_o of the input will be the actual resulting initial angle. In general, this is not true, since usnally the angular differences between finitely separated positions are preserved. Using MECSYN, the equivalent synthesis specifications

for point C'(x,y) are found by analyzing dyad ABC', represented by

$$X_j^{(k)}(t) = \frac{d^k}{dt^k}\{\overline{AB}\cos\theta + \overline{BC'}\cos\phi + X_a\}\Big|_{t=t_j} \qquad (3\text{-}1)$$

$$y_j^{(k)}(t) = \frac{d^k}{dt^k}\{\overline{AB}\sin\theta + \overline{BC'}\sin\phi + Y_a\}\Big|_{t=t_j} .$$

Step 2. Use the dynamic specs curvature transform of MECSYN as formulated in Chapter II with the resulting dynamic specs from Step 1, $\{X_j^{(k)}(t),\ y_j^{(k)}(t),\ \phi_j^{(k)}(t)\}$.

ℓ	j	k	$X_j^{(k)}(t)$	$y_j^{(k)}(t)$	$\phi_j^{(k)}(t)$
0	0	0	1.5 in	1 in.	0°
1	0	1	-5 in/sec	6 in/sec	1 rad/sec
2	0	2	-37 in/sec	-13 in/sec^2	4 rad/sec^2
3	0	3	-52 in/sec^3	-260 in/sec^3	8 rad/sec^3
4	1	0	0.5931 in.	1.5798 in.	20°

One of the solutions given by MECSYN is the originally chosen constraint link AB.

One of the remaining solutions is the constraint link C"D forming four-bar ABC"D shown in Fig. (3-1a). Although at first glance this appears to be a crank-rocker, MECSYN indicates it is a double-rocker mechanism with link length ratio 15.54.

C" = (-1.695, -1.301), D = (-1.535, -1.464).

Step 3. The angular cognate of dyad DC"B is calculated to give the angular cognate point, C = (1.16, .837). Point C completes the parallelogram DC"BC shown in Fig. (3-1a). Link DC now has the desired output angular dynamic specs. Figure (3-1b) shows the two prescribed finitely separated positions of the resulting four-link mechanism ABCD.

The verification for this problem was accomplished by analyzing the mechanism with IMP (Integrated Mechanisms Program) [25], which has been implemented at the University of Florida by the author. IMP is primarily the product of Dr. John Uicker, University of Wisconsin-Madison, and performs kinematic, static, dynamic, time response, equilibrium and vibration analyses for virtually any type of planar or spatial mechanism via a special user-oriented language.

The dynamic solutions are obtained by numerically integrating Hamilton's canonical equations which are automatically generated from the user's input statements describing the mechanical system.

IMP provides up to second order analyses (acceleration) which is true of most dynamic analysis programs. The mechanism has also been verified by the author's closed form, exact analysis program within MECSYN.

This analysis provides solutions up to the fourth order (jounce) to correspond with the synthesis capabilities but for reasons to be disclosed later in this chapter, only performs analysis for the specified precision conditions. IMP was used to provide a continuous analysis of the mechanism over the range of interest. Figure (3-2) illustrates

this analysis. The three curves are the position, velocity and acceleration of the output ($\phi_j^{(k)}(t)$) versus the position of the input ($\theta_j^{(k)}(t)$). Note that these curves are not plotted versus time but represent the output of the mechanism when the input has the velocity and acceleration prescribed at the first position. The specified precision conditions are circled in Fig. (3-2).

Again, this illustrates the difference between geometric kinematic synthesis and dynamic specs synthesis. These curves would be sufficient to describe the behavior of a kinematic function generator. Rather than time derivatives, they would be ϕ vs. θ, $d\phi/d\theta$ vs. θ and $d^2\phi/d\theta^2$ vs. θ. To completely describe the dynamic specs "function generator" two sets of graphs must be shown, i.e., output properties vs. time and input properties vs. time. Since many different time dependent input functions may be approximated by these chosen input specifications, the analysis cannot be completely defined for this example. Of course, in an applied problem, the input specifications are chosen to approximate the available time dependent input function, as will be demonstrated in Chapter IV. In many cases the available input will be close to constant speed. However, if the system power shaft has a known variation, dynamic specs synthesis has the capability of designing the mechanism for this input as illustrated by these examples.

As was stated previously, this method of four-bar "function-generator" synthesis leads to a second independent set of solutions. Step 1 and Step 2 are repeated exactly as before. Instead of Step 3, proceed as follows:

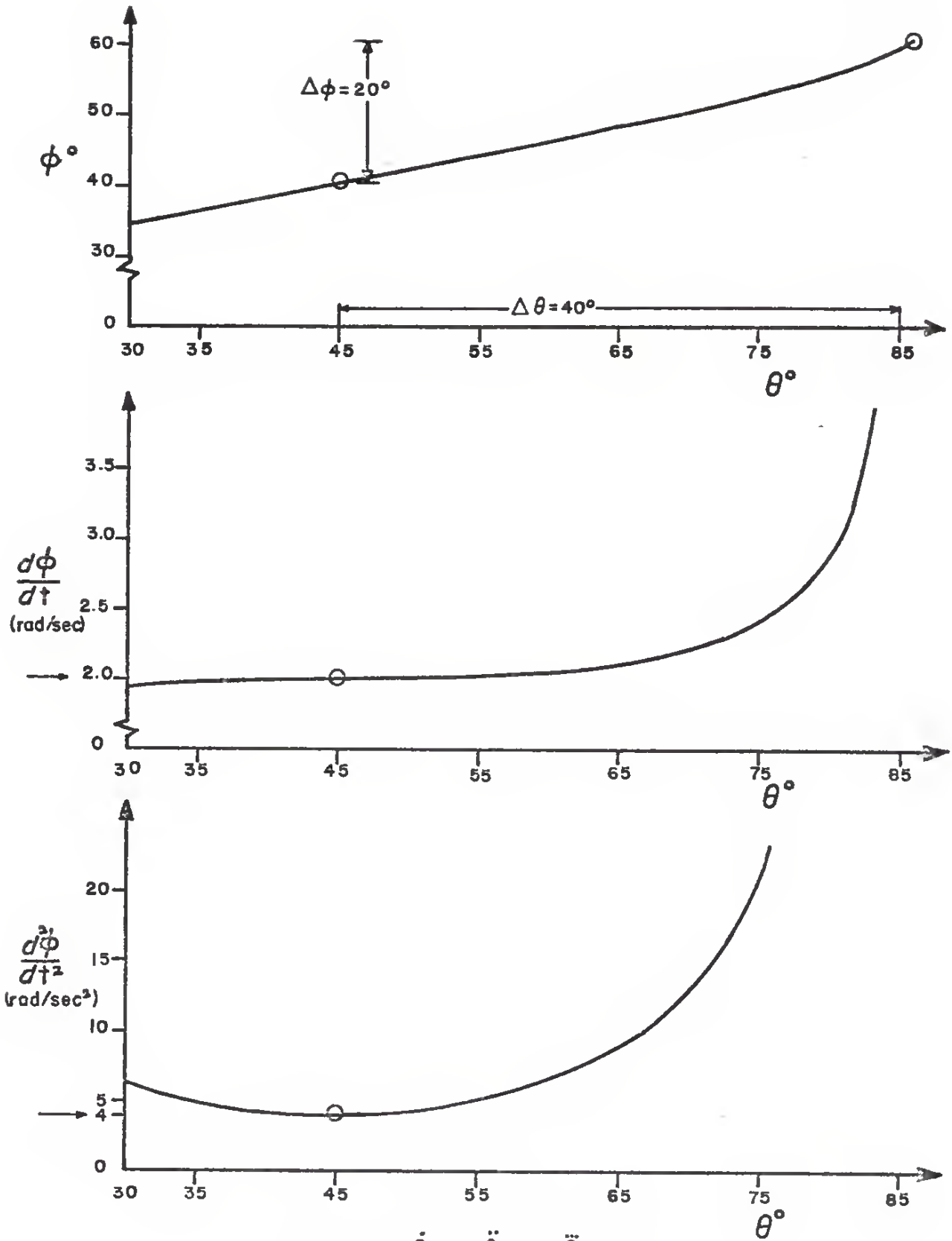


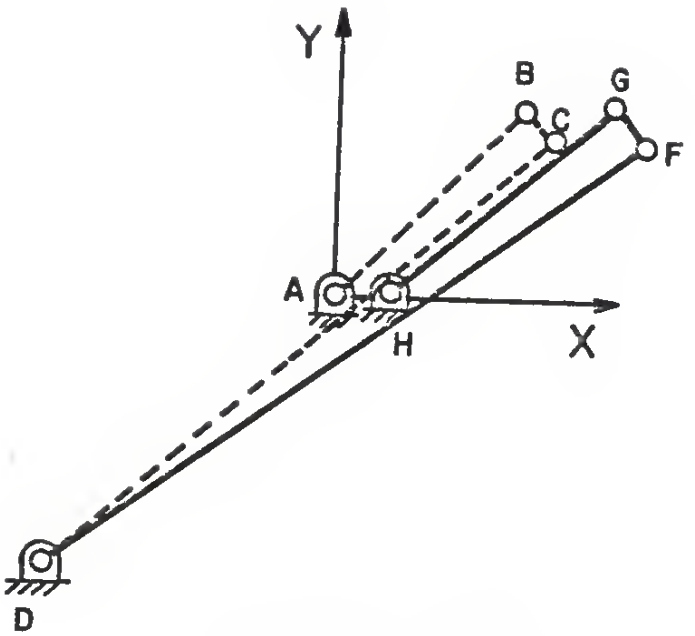
Figure 3-2 Four-Bar Output Link Properties

Step 3'. Step 2 resulted in a four-bar, ABC"D, whose coupler link BC'C" has the dynamic specs $\{ x_j^{(k)}(t),\ y_j^{(k)}(t),\ \phi_j^{(k)}(t) \}$, while input, AB has the specs $\theta_j^{(k)}(t)$. One might notice immediately from the analysis of the path cognates for the motion of coupler point C' in the MECSYN output that one of the path cognates has the output specs $\phi_j^{(k)}(t)$, while the input has the specs $\theta_j^{(k)}(t)$. This is due to the angular properties associated with path cognates as described in Chapter I. This path cognate mechanism is another solution to the dynamic specs four-bar "function generation" problem. The solution corresponding to the previously synthesized four-bar, ABC"D, is illustrated in Fig. (3-3) along with the "function generator" synthesized in Step 3 (ABCD) and is given by DFGH.

D = (-1.535, -1.464) F = (1.659, 0.837)

G = (1.498, 1.032) H = (0.299, 0.016)

These two four-bars have only point D in common and since they are "function generators" they may be moved anywhere in the plane and still serve their purpose. Therefore, they are independent solutions. For this example, in Step 2 there were four solutions for the constraint links, one of which was the chosen link AB. Thus, there were three possible solutions by angular cognates and three possible solutions by path cognates. As is the case in any mechanism synthesis problem, especially applied synthesis, many solutions must be rejected for various reasons, among them, dimensions, force transmission properties, closures, interference, physical compatibility of mechanism



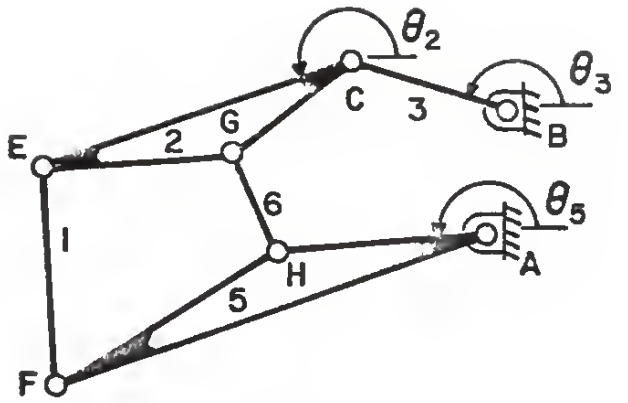
D=(-1.535,-1.464)
F=(1.659, 0.837)
G=(1.498, 1.032)
H=(0.299, 0.016)

Figure 3-3 Four-Bar Function Generator by Path Cognates

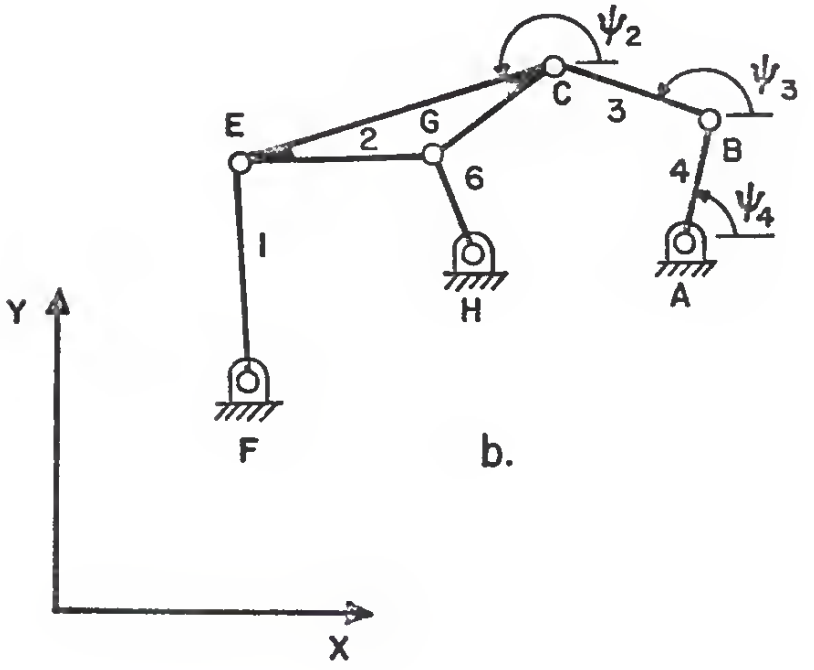
sections or loops, link length ratios, etc. For multi-link mechanisms, these problems are compounded but fortunately more solutions are usually available. It is desirable, then, to have as many solutions as possible without giving up any specification ability. This is especially true of dynamic specs synthesis, since it is necessary to specify at least one acceleration to treat dynamics. Although many of the six-link synthesis methods developed by the author [14] allow specification of 20 conditions, they are divided among two or more links. Also, most of these synthesis methods allow the choice of a parameter such as a pivot location or a link length which may be varied to obtain different solutions for optimization without affecting the maximum number of dynamic conditions specified. When this can be done, as in this four-bar example, it may be desirable to reduce the specification set to a four-position problem obtaining an infinity of solutions. All the synthesis methods are still valid. A four-position dynamic specs synthesis curvature transform has been included in the latest version of MECSYN which uses the same $D_{m\ell}$ coefficients and contains ordinary kinematic synthesis as a special case.

B. Synthesis of a Stephenson 2 Six-Link Mechanism to Coordinate the Angular Dynamic Specs of Three Links

A Stephenson 2 six-link mechanism (Fig. (3-4a)) will be synthesized to coordinate the angular dynamic specs of the output and a coupler with the input of Case 1 (PPPPP). This means specification of the angular velocity, acceleration, jerk and jounce for three links in one location. Either θ_3 or θ_5 may be the input. In this example, θ_3 will



a.



b.

Figure 3-4 Inversion of Stephenson 2 to Stephenson 3

be used as the input, since, in the author's experience, it is much more difficult to find a ternary input link (link 5) which will rotate full cycle, monotonically. The length of link 3 can be arbitrarily chosen thereby increasing the likelihood that it will satisfy this condition. That is not the intent in this example, but in many instances of practical significance, it may be a requirement. As for the coupler, either link 2, 1 or 6 may be chosen merely by using the path cognate feature of MECSYN in one design step. Since this problem involved specification of up to the fourth derivative with respect to time, it will be verified by the author's closed form analysis within MECSYN. This analysis will be illustrated along with the MECSYN synthesis steps.

Step 1. This problem will be solved by converting the specifications for the mechanism of Fig. (3-4a) into equivalent specifications for one of its inversions, the Stephenson 3 in Fig. (3-4b).

Prescribe the angular dynamic specs, $\{ {}^{(k)}\theta_{2j}(t), {}^{(k)}\theta_{5j}(5), {}^{(k)}\theta_{3j}(t)\}$, and the location of points A, B and C as in Fig. (3-4a).

ℓ	j	k	$^{(k)}\theta_{2j}(t)$	$^{(k)}\theta_{5j}(t)$	$^{(k)}\theta_{3j}(t)$	
0	0	0	0°	0°	90°	
1	0	1	6	2	5	rad/sec
2	0	2	7	4	10	rad/sec^2
3	0	3	2	8	15	rad/sec^3
4	0	4	3	8	18	rad/sec^4

A = (0, 0) B = (1, 0), C = (1, 1).

The actual initial angle of links 2 and 5 cannot be obtained by synthesis so they are nominally specified as zero.

Using the inversion feature of MECSYN with these specifications, the equivalent specifications are computed for point C and link 2 in Fig. (3-4b) from the following equations:

$$\psi_{mj} = \theta_{mj} - \theta_{5j} + \theta_{50} \qquad \text{where} \quad \overset{(k)}{\psi}_{mj} = \frac{d^k \psi_{mj}}{dt^k}, \text{ etc.}$$

$$\overset{(k)}{\psi}_{mj} = \overset{(k)}{\theta}_{mj} - \overset{(k)}{\theta}_{5j} \qquad m = 2,3,4 \quad j = 0 \quad k = 1,2,3,4. \tag{3-2}$$

These represent the inversion from the Stephenson 2 to the Stephenson 3:

$$\overset{(k)}{X}_j(t) = \frac{d^k}{dt^k}\{AB \cos \psi_4 + BC \cos \psi_3 + X_A\}\Big|_{t=t_j}$$

$$\overset{(k)}{Y}_j(t) = \frac{d^k}{dt^k}\{AB \sin \psi_4 + BC \sin \psi_3 + Y_A\}\Big|_{t=t_j} \tag{3-3}$$

which are the equivalent specifications for point C in the Stephenson 3 inversion of Fig. (3-4b).

The designer need not be concerned with these equations since MECSYN carries out these steps automatically. The original and the equivalent specifications are illustrated in Table 3-1 from MECSYN.

Step 2. Using the equivalent inverted dynamic specifications, $\{\overset{(k)}{X}_j(t), \overset{(k)}{Y}_j(t), \overset{(k)}{\psi}_{2j}(t)\}$, MECSYN continues to find two constraint links (1 and 6) as in Fig. (3-4b) with the dynamic specs curvature transform of Chapter II. The converted specifications and the solutions are shown in

Table 3-2. Links 1 and 6 or EF and GH are given by

E = (0.865, 0.810), F = (7.031, -4.621)

G = (0.698, -0.683), H = (2.309, 1.191).

Table 3-3 shows the solution again for the four-bar EFGHC (Fig. (3-4b)) along with its two path cognates, Grashof's mobility criteria, link length ratios and the analysis of all three mechanisms.

As was mentioned previously, if $\theta_{6j}^{(k)}(t)$ or $\theta_{1j}^{(k)}(t)$ was specified instead of $\theta_{2j}^{(k)}(t)$, one of the path cognates illustrated in Table 3-3 would be chosen in place of the original mechanism EFGH (or AO_ABO_B in Table 3-3).

Step 3. The Stephenson 3 mechanism EFGHCBA synthesized in Step 2 is inverted by releasing link FHA and fixing link AB. The result is a Stephenson 2 mechanism which satisfies the desired specifications. This mechanism is shown in Fig. (3-5).

In order to provide analysis verification for examples of this type, the author has considered several types of analysis methods to be included as an integral part of MECSYN. Although many different analysis methods are available, they were not suitable for various reasons. The desired analysis routine should have the following properties: (a) in keeping with the closed form nature of MECSYN it should be a closed form solution (non-iterative), (b) it should provide solutions for up to fourth order derivatives, (c) it should be equally applicable to all types of mechanisms capable of being synthesized by MECSYN, and

Table 3-1 Equivalent Inverted Synthesis Specifications

INPUT PARAMETER CONVERSION FOR SYNTHESIS
OF SIX-LINK AND CEARED FIVE-LINK MECHANISMS
(BASED ON STEPHENSON-2 SIX-LINK MECHANISM)

CASE NUMBER 1

POINT A	POINT B	POINT C
(0.0 , 0.0)	(1.0000, 0.0)	(1.0000, 1.0000)

LINK 3		LINK 4	
LENGTH	ANGLE	LENGTH	ANGLE
1.0000	90.0000	1.0000	0.0

		STEPHENSON-2			STEPHENSON-3	
POS.	DER.O	THETA 2	THETA 3	THETA 5	PSI 3	PSI 4
0	0	0.0	90.0000	0.0	90.0000	0.0
0	1	6.0000	5.0000	2.0000	3.0000	-2.0000
0	2	7.0000	10.0000	4.0000	6.0000	-4.0000
0	3	2.0000	15.0000	8.0000	7.0000	-8.0000
0	4	3.0000	18.0000	8.0000	10.0000	-8.0000

		CONVERTED SPECIFICATIONS FOR SYNTHESIS		
POS.	DER.	XC	YC	PSI 2
0	0	1.00000	1.00000	0.0
0	1	-3.00000	-2.00000	4.00000
0	2	-10.00000	-13.00000	3.00000
0	3	-4.00000	-54.00000	-6.00000
0	4	218.00000	-23.00000	-5.00000

Table 3-2 Constraint Link Synthesis

```
*****************************************************
*                      MECSYN                       *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
*****************************************************
```

OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	YES
INTERMEDIATE PRINTING	NO
ANALYSIS	YES
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	NO
FIVE- AND SIX-LINK SYNTHESIS	YES
FOUR OR FIVE MSP (45=BOTH)	5

CONVERTED SPECIFICATIONS FOR SYNTHESIS

POS.	DER.	XC	YC	PSI 2
0	0	1.00000	1.00000	0.0
0	1	-3.00000	-2.00000	4.00000
0	2	-10.00000	-13.00000	3.00000
0	3	-4.00000	-54.00000	-6.00000
0	4	218.00000	-23.00000	-5.00000

COORDINATES OF THE CONSTRAINT LINKS

POINTS				CENTERS			
X =	0.86466	Y =	0.80954	X =	7.03110	Y =	-4.62124
X =	0.69785	Y =	-0.68349	X =	2.30856	Y =	1.19096

Table 3-3 Cognates and Analysis

COUPLER POINT
(1.000, 1.000)

ORIGINAL MECHANISM			FIRST COGNATE			SECOND COGNATE		
A(	0.865,	0.810)	O(	7.166,	-4.431)	F(	2.611,	2.674)
OA(	7.031,	-4.621)	OO(	7.031,	-4.621)	OF(	2.309,	1.191)
B(	0.898,	-0.663)	E(	7.595,	-5.635)	G(	0.840,	0.887)
OB(	2.309,	1.191)	OF(	7.235,	-5.778)	OG(	7.235,	-5.776)

DOUBLE-ROCKER		CRANK-ROCKER		CRANK-ROCKER	
LINK LENGTH RATIO	5.47	LINK LENGTH RATIO	5.47	LINK LENGTH RATIO	5.47
MODIFIED L.L.R.	8.45	MODIFIED L.L.R.	37.60	MODIFIED L.L.R.	F.55

ANALYSIS OF ANGULAR PROPERTIES

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	138.62962	135.39936	219.42691	54.60207	296.78010	79.82512
0	1	0.41213	0.43898	-3.14869	4.00000	-1.68212	4.00000
0	2	1.53027	2.25906	0.78935	3.00000	2.75370	3.00000
0	3	4.51256	4.75719	15.26975	-F.00000	-25.40402	-F.00000
0	4	-3.81661	-21.98996	-20.60857	-5.00000	-151.30300	-5.00000
		LINK LENGTH	8.21696	LINK LENGTH	0.23365	LINK LENGTH	1.71030

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	234.80207	39.42F91	315.39936	318.62962	327.27753	49.327F0
0	1	4.00000	-3.14889	0.43898	0.41213	0.34585	1.99203
0	2	3.00000	0.78935	2.25908	1.53027	9.22500	-3.47330
0	3	-6.00000	15.26975	4.75719	4.51256	2.0F507	-33.46909
0	4	-5.00000	-20.80657	-21.96996	-3.81661	-8F.04654	-70.25F4F
		LINK LENGTH	0.23365	LINK LENGTH	F.21698	LINK LENGTH	2.47143

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	229.32769	147.27753	244.99892	20.30458	134.82956	130.47394
0	1	1.99203	0.34585	-1.2179F	1.99203	0.41213	0.3F105
0	2	-3.47130	9.22500	6.88969	-3.47130	1.53027	1.88812
0	3	-33.46909	2.06507	41.90565	-33.46909	4.51256	3.92401
0	4	-70.25F46	-86.04654	49.19569	-70.25646	-3.81661	-17.2441F
		LINK LENGTH	2.47143	LINK LENGTH	0.3843F	LINK LENGTH	9.35497

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	259.82512	116.78010	310.47394	314.62956	200.30458	64.99892
0	1	4.00000	-1.68212	0.3F195	0.41213	1.99203	-1.2179F
0	2	3.00000	2.75370	1.88812	1.53027	-3.47130	F.88969
0	3	-6.00000	-25.40402	3.92401	4.51256	-33.46909	41.90565
0	4	-5.00000	-151.30300	-17.24416	-3.81661	-70.25F4F	49.19569
		LINK LENGTH	1.71039	LINK LENGTH	9.35497	LINK LENGTH	0.38438

LINK A-B	1.50233	LINK O-E	1.27792	LINK F-G	2.81771
LINK OA-OB	7.48893	LINK OO-OF	1.1F470	LINK OF-OG	8.52F11

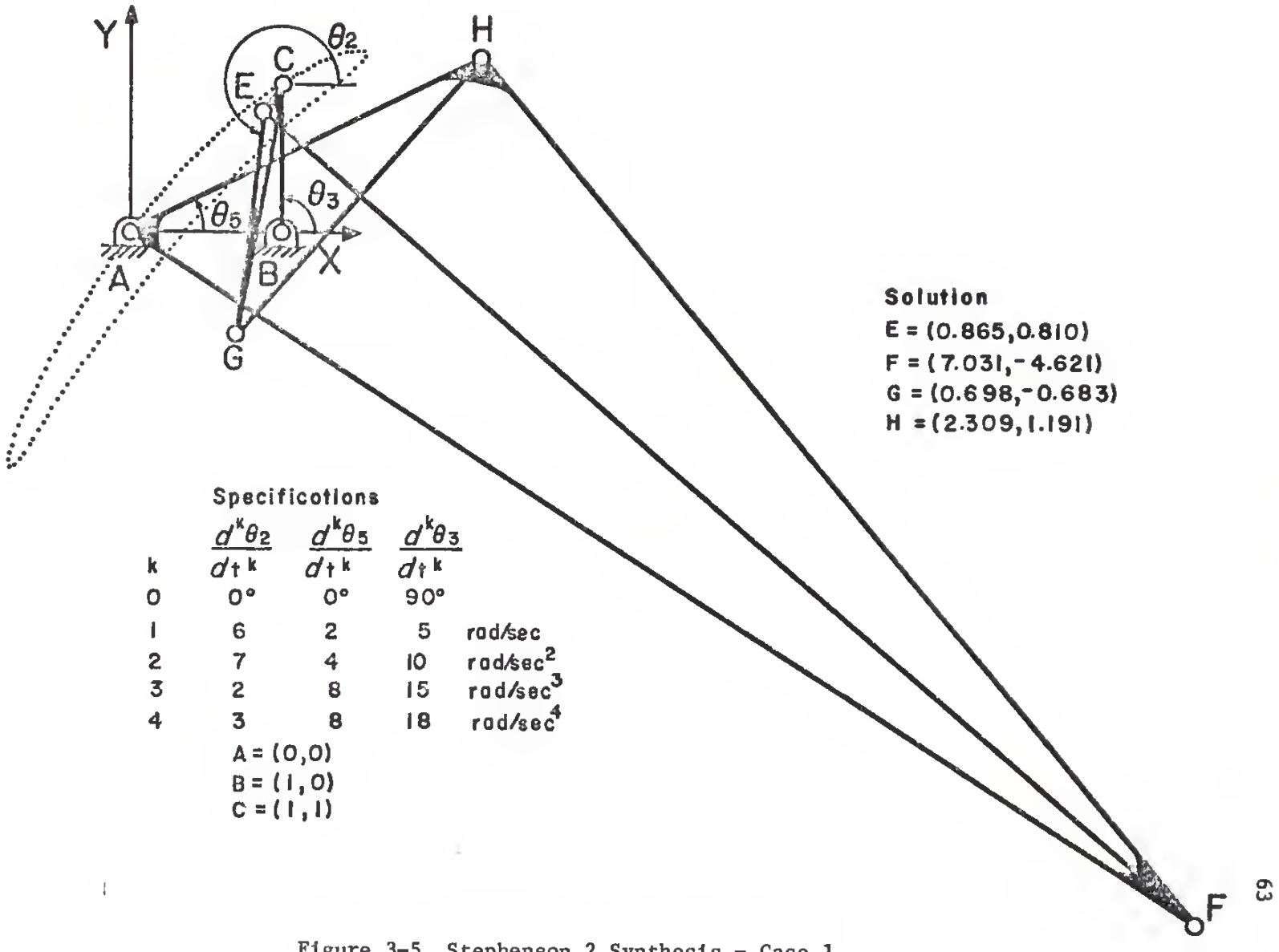


Figure 3-5 Stephenson 2 Synthesis - Case 1

(d) since the complete input cycle is not always known, it should provide solutions for mechanisms at the precision points for the order of derivatives specified automatically by case number.

The routine developed to satisfy these conditions does not overshadow the rest of the MECSYN in size and yet provides enough information about the specified conditions to verify the mechanism and provides analysis of similar properties for all other links of the mechanism. The designer can observe how other links of the mechanism are behaving at the precision locations in addition to verifying the solution. If additional full cycle information is desired, IMP or the dyad package analysis [26] may be used. This analysis routine serves only the needs of the synthesis process.

Since most analysis programs approach each different mechanism in a different fashion, this type of analysis was prohibitive in size. The routine to be described here follows the modular synthesis approach of MECSYN.

Figure (3-6) shows the path cognates of an original set of two dyads, CAO_A and CBO_B which are partial results of a synthesis problem. The remaining four dyads represent the first and second cognate solutions of MECSYN (Table 3-3). The locations of all these points for the first specified position are known after a synthesis and cognate calculation step of MECSYN. Depending on the case number, a series of vectors for position, velocity, acceleration, jerk or jounce is applied to point C which consists of all the path specifications for point C in the synthesis step (Fig. (3-6)). For the synthesis step to be valid,

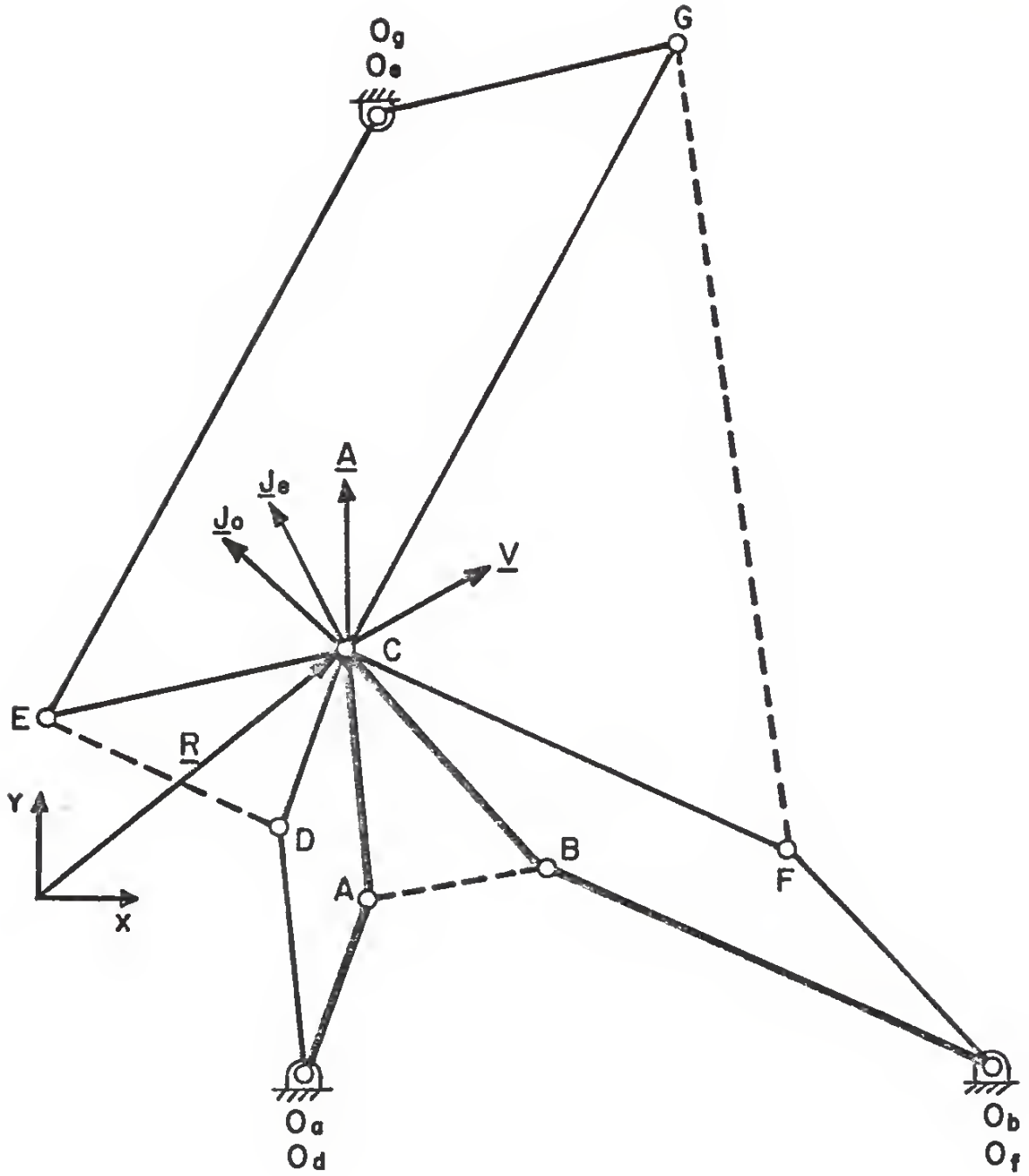


Figure 3-6 Analysis of a Four-Bar and Its Path Cognates

coupler links formed by AC and BC must have identical angular properties equal to the angular properties $\phi_j^{(k)}(t)$ specified for the coupler link when point C has the specified state $\{x_j^{(k)}(t), y_j^{(k)}(t)\}$. If this is the case, the two dyads form a valid four-bar solution for the synthesis problem and dimension AB is constant. Also, note each cognate dyad constraint link DO_d and FO_F must have precisely the same angular properties as those specified for the coupler. These corresponding properties are enclosed in rectangles in Table 3-2 and Table 3-3. Other similarities between the angular properties of links in path cognates as described in Chapter I may also be observed in Table 3-3.

Referring to Fig. (3-7), the analysis is formulated as follows: The location of the points A and O_A in the first position for all the dyads of Fig. (3-6) is determined by the synthesis and path cognate steps. The dynamic specifications $\{x_{cj}^{(k)}(t), y_{cj}^{(k)}(t)\}$ for the coupler point C are represented in Fig. (3-7) by $C_j^{(k)}(X_2,Y_2)$. Thus the known quantities are A, O_A and $\{x_{cj}^{(k)}(t), y_{cj}^{(k)}(t)\}$.

(X_1,Y_1) are the coordinates of O_A and (X_2,Y_2) are the coordinates of the first position of C. The desired properties are

$$\psi_{1j}^{(k)}(t) \text{ and } \psi_{2j}^{(k)}(t)$$

for both closures of each dyad (whether point A is above or below line O_AC). Both closures are necessary since frequently one or more dyads may change into another closure while satisfying the prescribed conditions. This does not mean that the mechanism will have closure difficulties in satisfying the specifications. For example, the four-bar O_AACBO_B of Fig. (3-6) will have closure difficulties if dyad ABO_B

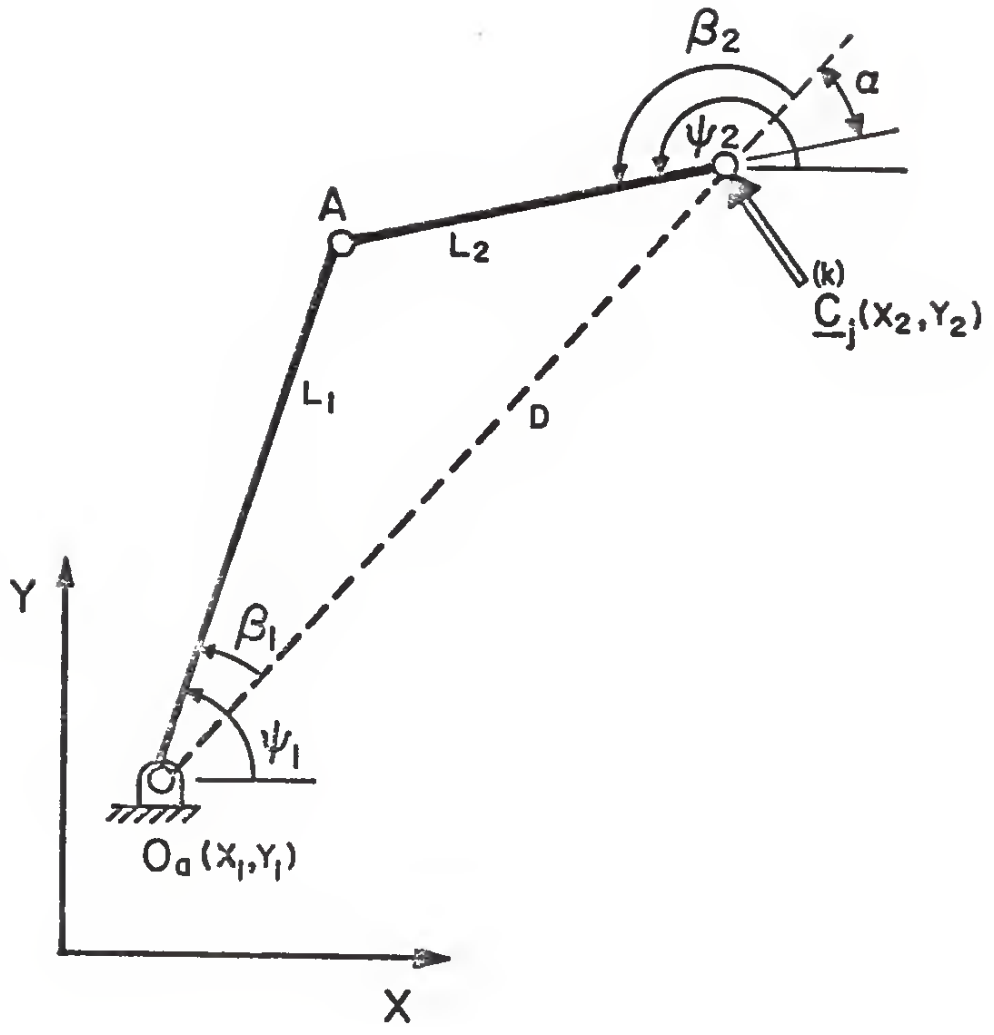


Figure 3-7 Dyad Analysis

must go into a different closure when AO_A is the input, regardless of what closures the dyads CAO_A and CBO_B are going into.

This closure problem is more difficult to define for six-bars in terms of dyad closures. It appears that a test for closures must be written independently for each mechanism rather than relying on a modular approach as is done for all other components of MECSYN. Solution of this closure problem is necessary to provide complete computer selection of optimum mechanisms. If another analysis routine is used such as IMP or the dyad package, it will be immediately obvious whether a mechanism satisfies all prescribed conditions in one closure since these analyses are performed for each mechanism separately and in the original closure. This approach would be prohibitive, costwise, for evaluation of a large number of synthesized mechanisms to obtain optimum solutions.

The angles ψ_1 and ψ_2 in Fig. (3-7) are given in the finitely separated positions by:

$$\psi_{1j} = \alpha_j \pm \beta_{1j} \qquad \psi_{2j} = \alpha_j \pm \beta_{2j}$$

where $\pm$ determines the closure and

$$0 \leq \beta_1 \leq \Pi \qquad 0 \leq \beta_2 \leq \Pi \tag{3-4}$$

$$D^2 = (X_{2j}-X_1)^2 + (Y_{2j}-Y_1)^2$$

$$\beta_1 = \cos^{-1}\left[(L_1^2 - L_2^2 + D^2)/2L_1D\right] \qquad L_1 = |AO_A|$$

$$\beta_2 = \cos^{-1}\left[(L_1^2 - L_2^2 - D^2/2L_2D\right] \qquad L_2 = |AC|$$

$$D = |O_AC| \ .$$

The vectors $\underline{\psi}$ and $\underline{r}$ and the matrix $\mathbf{A}$ are defined by:

$$\underline{\psi} = \begin{bmatrix} \psi_{1j}(t) \\ \psi_{2j}(t) \end{bmatrix} \qquad \underline{r} = \begin{bmatrix} X_{2j}(t) - X_1 \\ Y_{2j}(t) - Y_1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -L_1 \sin \psi_{1j}(t) & L_2 \sin \psi_{2j}(t) \\ L_1 \cos \psi_{1j}(t) & -L_2 \cos \psi_{2j}(t) \end{bmatrix} .$$

All the zeroth order properties, $\psi_{1j}(t)$, $\psi_{2j}(t)$, are given by equation (3-4). The higher order properties,

$$\frac{d^k \psi_{1j}(t)}{dt^k} \quad \text{and} \quad \frac{d^k \psi_{2j}(t)}{dt^k} ,$$

are determined from the following equations (where $\dot{\underline{\psi}}$ represents $d\underline{\psi}/dt$, etc.):

$$\begin{aligned} \dot{\underline{\psi}} &= \mathbf{A}^{-1} \dot{\underline{r}} \\ \ddot{\underline{\psi}} &= \mathbf{A}^{-1} [\ddot{\underline{r}} - \dot{\mathbf{A}} \dot{\underline{\psi}}] \\ \dddot{\underline{\psi}} &= \mathbf{A}^{-1} [\dddot{\underline{r}} - 2 \dot{\mathbf{A}} \ddot{\underline{\psi}} - \ddot{\mathbf{A}} \dot{\underline{\psi}}] \\ \ddddot{\underline{\psi}} &= \mathbf{A}^{-1} [\ddddot{\underline{r}} - 3 \dot{\mathbf{A}} \dddot{\underline{\psi}} - 3 \ddot{\mathbf{A}} \ddot{\underline{\psi}} - \dddot{\mathbf{A}} \dot{\underline{\psi}}] . \end{aligned} \tag{3-5}$$

A representative example (case 1) of this analysis is shown in Table 3-3. The properties $\psi_{1j}^{(k)}(t)$ correspond to the columns labeled constraint link and the properties $\psi_{2j}^{(k)}(t)$ to those columns labeled coupler link. The columns of the analyses correspond to the mechanisms listed directly above them. More detailed information about the use of this analysis routine may be found in the description of MECSYN to follow (under the heading subroutine ANALYS).

For the previous Stephenson 2 example, Tables 3-2 and 3-3 give verification of the inverted four-bar which satisfies the inverted specifications for point C. What remains to be verified are the inverted specifications for point C, of the dyad ABC in Fig. (3-4b).

However, this can be immediately verified as was done in Section A for the four-bar "function generator." That is, choosing arbitrarily a dyad ABC and the angular specifications desired for this dyad, $\psi_{4j}^{(k)}(t)$, $\psi_{3j}^{(k)}(t)$ as in Fig. (3-4b). MECSYN is used to find the equivalent specifications for point C. Synthesizing constraint links for this dyad using the specifications $\{ X_{cj}^{(k)}(t), y_{cj}^{(k)}(t), \psi_{3j}^{(k)}(t)\}$ gives two or four constraint links in general. For the computed specifications of point C to be correct, one of these solutions must be the arbitrarily chosen constraint link AB. In this fashion the routine that computes the inverted specifications of point C has been verified for all seven cases.

Thus there are three independent self-checking routines within MECSYN: the routine which computes the inverted specifications for point C of a chosen dyad ABC, the dynamic specs synthesis curvature transform and the analysis routine described previously in this chapter which computes angular properties given the path specifications. Since these verification techniques are applied modularly just as the synthesis procedure is applied to a mechanism, any mechanism that can be synthesized can be immediately verified. An additional benefit arises from this; i.e., if any errors are made in specification during the design steps, the errors are quickly revealed by comparing these self-checking features.

C. Dynamic Specs Synthesis of a Geared Five-Link Mechanism

To illustrate the versatility of MECSYN a geared five-link mechanism will be designed for prescribed dynamic specs of coupler link 2 with input link 5 for Case 4 (PPP-P-P) as illustrated in Fig. (3-8a). This implies specification of velocities and accelerations at the first position and two additional finite positions. Some preliminary formulations are necessary for the design of the geared five-bar. The gear ratios as shown in Fig. (3-8a) are:

$$\left.\begin{aligned} M &= r_a/r_b \\ N &= r_b/r_c \end{aligned}\right\} \text{ with idlers} \qquad \left.\begin{aligned} M &= -r_a/r_b \\ N &= -r_b/r_c \end{aligned}\right\} \text{ without idlers}$$

$Q = MN$ = gear ratio between points A and C.

The angular relationships between links 2, 3 and gear B are

$$\Delta\theta_2 = N(\Delta\theta_B - \Delta\theta_3) + \Delta\theta_3$$

$$\Delta\theta_B = M\,\Delta\theta_5$$

$$\Delta\theta_2 = N(M\,\Delta\theta_5 - \Delta\theta_3) + \Delta\theta_3$$

$$\Delta\theta_2 = Q\,\Delta\theta_5 + (1-N)\,\Delta\theta_3 \quad .$$

The higher order angular properties are obtained by

$$\frac{d^k}{dt^k}(\theta_{2j}-\theta_{20}) = \frac{d^k}{dt^k}\left[Q(\theta_{5j} - \theta_{50}) + (1-N)(\theta_3 - \theta_{30})\right]$$

or

$$\dot{\theta}_2 = Q\,\dot{\theta}_5 + (1-N)\dot{\theta}_3$$

$$\ddot{\theta}_2 = Q\,\ddot{\theta}_5 + (1-N)\ddot{\theta}_3 \quad .$$

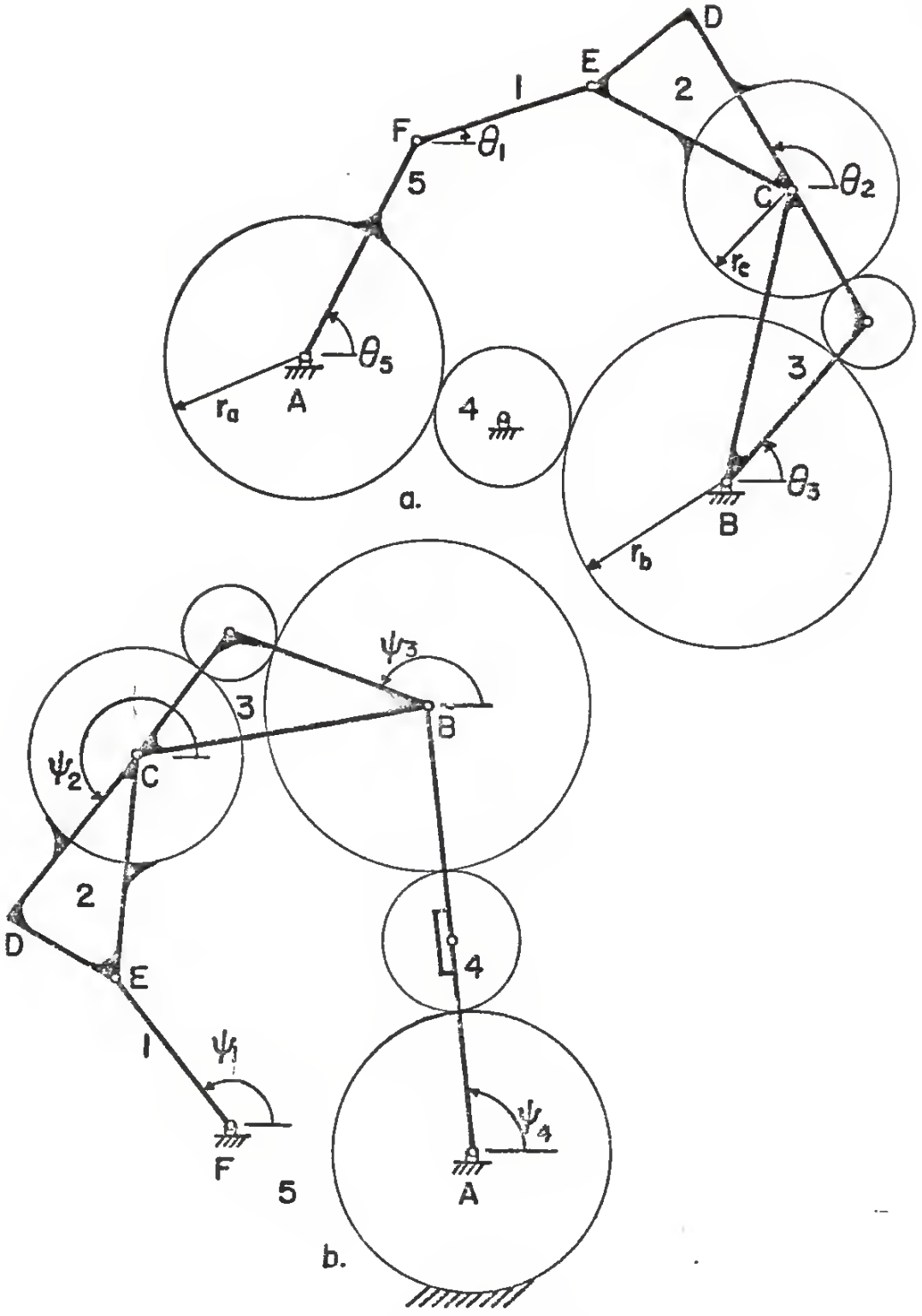


Figure 3-8 Geared Five-Bar Inversion

Thus the relation of the input link 5 to links 2 and 3 is given by

$$\Delta\theta_5 = \frac{1}{Q}\{\Delta\theta_2 - (1-N)\Delta\theta_3\}$$

$$\dot{\theta}_5 = \frac{1}{Q}\{\dot{\theta}_2 - (1-N)\dot{\theta}_3\} \qquad (3\text{-}6)$$

$$\ddot{\theta}_5 = \frac{1}{Q}\{\ddot{\theta}_2 - (1-N)\ddot{\theta}_3\} \quad .$$

A constant speed input will be selected, i.e., $\ddot{\theta}_5 = 0$, $\dot{\theta}_5 = 0.1$ rad/sec. Two free parameters are available which allow solution of this problem for dynamic state specifications, namely, the gear ratios M and N. In the case of geometric kinematic synthesis, the specified motion of link 2 would be achieved, regardless of what input, θ_5, is supplied. If, however, it is desired that coupler link 2 satisfy a prescribed inertia torque and force, a specific velocity and acceleration must be supplied at the input, θ_5. The necessary input may be found by analysis after synthesis and provided, but it is not usually feasible to provide an arbitrary acceleration and velocity to the input. Instead the mechanism should be synthesized to be driven with the available velocity and acceleration of the power shaft. The parameters M and N are determined in the synthesis procedure to allow the desired input velocity and acceleration.

N is chosen to give the input acceleration condition at position zero, $\ddot{\theta}_5 = 0$.

$$0 = \frac{1}{Q}\{\ddot{\theta}_2 - (1-N)\ddot{\theta}_3\}$$

$$N = 1 - (\ddot{\theta}_2/\ddot{\theta}_3)\,. \qquad (3\text{-}7)$$

Q is chosen to coordinate the desired input velocity $\dot{\theta}_5 = 0.1$ rad/sec, with the specifications of link 2.

$$\dot{\theta}_5 = \frac{1}{Q}\{\dot{\theta}_2 - (1-N)\dot{\theta}_3\}$$

$$Q = \frac{1}{\dot{\theta}_5}\{\dot{\theta}_2 - (1-N)\dot{\theta}_3\}$$

$$Q = 10\{\dot{\theta}_2 - (\ddot{\theta}_2/\ddot{\theta}_3)\dot{\theta}_3\} \quad . \tag{3-8}$$

The properties $\overset{(k)}{\theta}_{2j}(t)$ are prescribed and the properties $\overset{(k)}{\theta}_{3j}(t)$ arise from the analysis routine in MECSYN after the first synthesis step. N and Q can then be found.

Step 1. Prescribe the dynamic specs of link 2, $\{\overset{(k)}{X}_{Dj}(t), \overset{(k)}{Y}_{Dj}(t), \overset{(k)}{\theta}_{2j}(t)\}$, tht velocity $\dot{\theta}_5$ and the acceleration $\ddot{\theta}_5$ of the input and the location of point A (Fig. (3-8a)).

ℓ	j	k	$\overset{(k)}{X}_{Dj}(t)$	$\overset{(k)}{Y}_{Dj}(t)$	$\overset{(k)}{\theta}_2(t)$
0	0	0	2 in.	2 in.	0°
1	0	1	2 in./sec	2 in./sec	4 rad/sec
2	0	2	1 in./sec	-3 in./sec^2	6 rad/sec^2
3	1	0	2.5 in.	2.3 in.	10°
4	2	0	3 in.	2.5 in.	15°

$\dot{\theta}_5 = 0.1$ rad/sec

$\ddot{\theta}_5 = 0$ rad/sec^2

A = (0, 3).

Using the dynamic specs curvature transform of MECSYN, a suitable constraint link BC is found and shown in Fig. (3-9)

$$B = (1.248, 3.076) \qquad C = (0.755, 4.200).$$

The values of $\theta_{3j}^{(k)}(t)$ for this link are obtained from MECSYN's analysis.

Step 2. The synthesis is carried out similar to that of a Stephenson 2 mechanism. Referring to Fig. (3-8), link 5 will be fixed and link 4 released. The inverted properties of dyad ABC $(\psi_{4j}^{(k)}(t), \psi_{3j}^{(k)}(t))$ will be found from $\theta_{5j}^{(k)}(t)$, $\theta_{2j}^{(k)}(t)$ and $\theta_{3j}^{(k)}(t)$. Using the inverted dynamic specs $\{X_{Cj}^{(k)}(t), Y_{Cj}^{(k)}(t), \psi_{2j}^{(k)}(t)\}$,

a constraint link EF will be found as shown in Fig. (3-8b).

For the geared five-bar some intermediate calculations must be carried out first.

From the MECSYN analysis

ℓ	j	k	$\theta_{2j}^{(k)}(t)$	$\theta_{3j}^{(k)}(t)$
0	0	0	119.49271°	113.64762
1	0	1	4.0 rad/sec	6.05118 rad/sec
2	0	2	6.0 rad/sec	9.17404 rad/sec^2
3	1	0	129.49271°	106.83605°
4	2	0	134.49271°	90.90280°

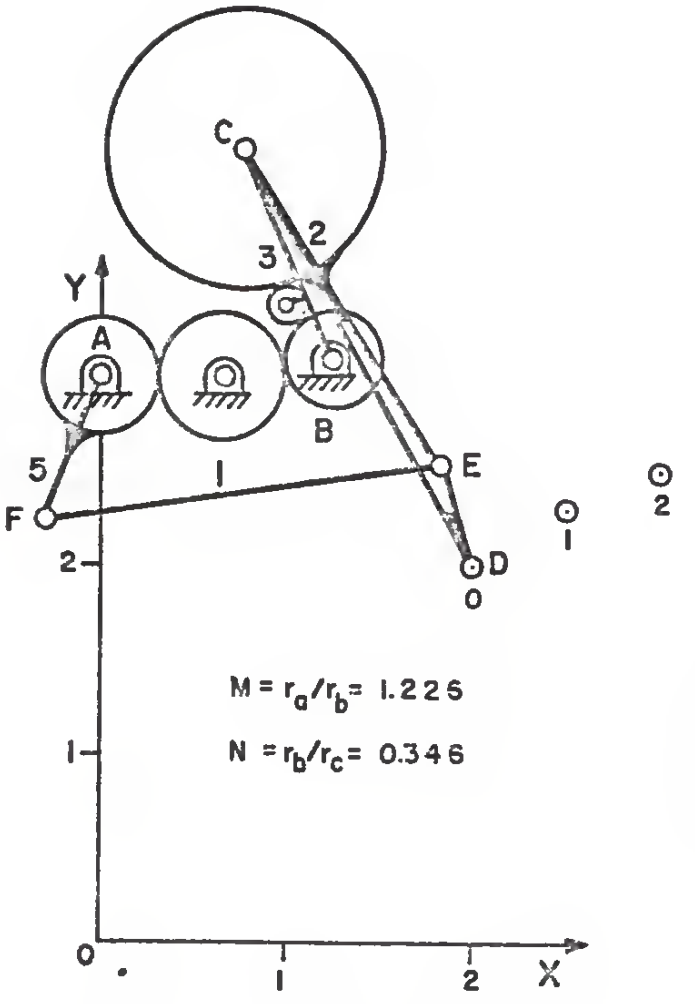
Using Eqs.(3-7) and (3-8) and the above, N and Q can be calculated as

$$N = 0.34598, \qquad Q = 0.43408, \qquad M = Q/N = 1.22574.$$

Specifications

pos.	der.	X_D	Y_D	θ_2	θ_5
0	0	2	2	0°	
0	1	2	2	4	0.1
0	2	1	-3	6	0
1	0	2.5	2.3	10°	
2	0	3.0	2.5	15°	

A = (0,3)



A = (0,3)
B = (1.248, 3.076)
C = (0.755, 4.200)
D = (2,2)
E = (1.816, 2.523)
F = (-0.307, 2.249)

Figure 3-9 Synthesis of Geared Five-Bar - Position 0

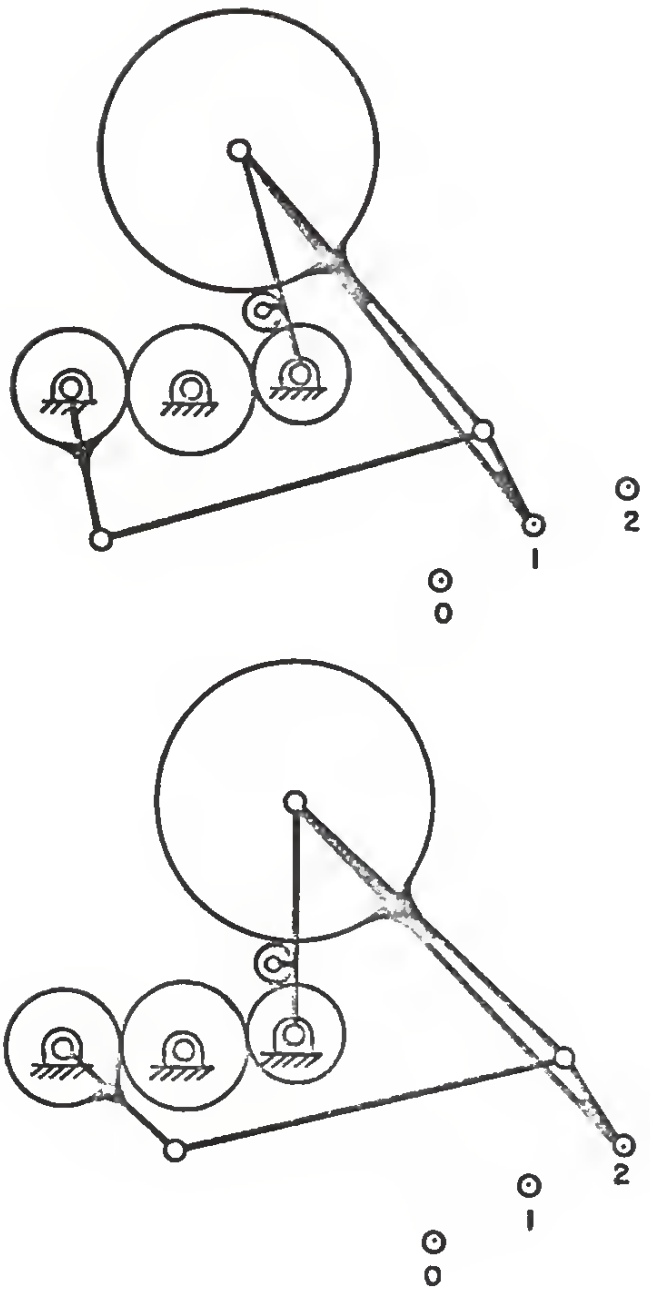


Figure 3-10 Synthesis of Geared Five-Bar - Positions 1, 2

Applying Eq. (3-6) will give the angles, θ_{5j},

$$\Delta\theta_5 = \theta_{5j} - \theta_{50} = \frac{1}{Q}\,[\Delta\theta_2 - (1-N)\Delta\theta_3]$$

$$(\theta_{51} - \theta_{50}) = \Delta\theta_{51} = 34.08532°$$

$$(\theta_{52} - \theta_{50}) = \Delta\theta_{52} = 70.44795° .$$

Using these specifications

ℓ	j	k	$\theta_{5j}^{(k)}(t)$
0	0	0	0°
1	0	1	0.1 rad/sec
2	0	2	0 rad/sec^2
3	1	0	34.08532°
4	2	0	70.44795°

along with the previously listed specifications for $\theta_{2j}^{(k)}$ and $\theta_{3j}^{(k)}$ with MECSYN, the inverted specification set $\{X_{Cj}^{(k)}(t),\ y_{Cj}^{(k)}(t),\ \psi_{2j}^{(k)}(t)\}$ is found.

The angular properties $\psi_{mj}^{(k)}(t)$ of Fig. (3-8b) are related to $\theta_{mj}^{(k)}(t)$ of Fig. (3-8a) by

$$\psi_{mj} = \theta_{mj} - \theta_{5j} + \theta_{50} \qquad m = 1,\ldots,5$$

$$\psi_{mj}^{(k)} = \theta_{mj}^{(k)} - \theta_{5j}^{(k)} \qquad \text{for } k > 0.$$

Step 3. Using the inverted specification set for the dynamic specs of link 2 in Step 2, a constraint link EF is synthesized with MECSYN.

ℓ	j	k	$X_{Cj}^{(k)}(t)$	$y_{Cj}^{(k)}(t)$	$_{Cj}^{(k)}(t)$
0	0	0	0.7554 in.	4.2005°	119.4927°
1	0	1	-6.6820 in./sec	-3.0540 in./sec	3.9 rad/sec
2	0	2	7.1076 in./sec^2	-44.3275 in./sec^2	6.0 rad/sec^2
3	1	0	1.4400 in.	3.5360 in.	95.4070°
4	2	0	1.6393 in.	2.2788 in.	64.0448°

E = (1.816, 2.523) F = (-0.307, 2.249).

Step 4. Reinvert the mechanism by fixing link 4 and releasing link 5. Applying the gearing M, N from point A to C gives the desired geared five-bar mechanism ABCDEF shown in Figs. (3-9) and (3-10).

The following chapter presents solutions to two practical problems using the methods of Chapter II and this chapter.

CHAPTER IV

EXTENDED PRACTICAL APPLICATION OF DYNAMIC SPECS SYNTHESIS

The theory of Chapter II is applied in this chapter to the solution of two problems. First, the dynamic specs of a link moving with general coplanar motion and the corresponding dynamic input specs are prescribed to synthesize a frequently needed transfer device between linear and circular moving conveyors. Both a Stephenson 3 and a Stephenson 2 six-link mechanism are synthesized for the same transfer specifications. Second, a Stephenson 3 six-link mechanism is synthesized for torch or bandsaw cutting of large, low pitch (diametral pitch = 3/8) gear teeth. The geometric specs of a coupler link (path and angular properties) are coordinated with the corresponding geometric input.

Today the designer is faced with a much wider range of systems capable of solving any given mechanical motion problem. One end of this spectrum is represented by automatic control devices composed of electronic, pneumatic, or hydraulic components. The other end is dominated by single degree of freedom systems such as cams and linkages. The spectrum has been dramatically enlarged during the past three decades primarily due to the increased activity in the design and application of automatic control devices. This led to a relative de-emphasis of study and research in the mechanism field limiting the flexibility

with which the designer can meet any given problem. The objective of most synthesis procedures is to re-establish mechanisms as an effective and viable alternative to systems based solely on control type components.

Mechanisms are generally considered devices with limited adjustability capable of performing a given collection of functions with high precision and reliability. Unless internal stresses and energy storage lead to vibration and consequent destruction, these systems can be operated at high speeds, can drive large masses, and overcome large forces. The cam is the most flexible information storage device among mechanisms but it is the least capable of fulfilling the above requirements because of its structural weakness. Linkages can be made to handle very high loads but they can only meet a limited range of motion functions. Consequently, designers of linkages must understand the requirements of their output functions to a greater degree and be very careful not to overspecify the problem because the linkage has a limited number of free design parameters.

These limitations of linkages have prevented their broad use in recent years. Design procedures as presented here are intended to strengthen the designer's capability in creating a mechanism to satisfy a needed function or group of functions. Utilizing dynamic specs synthesis linkages can now be asked to perform combinations of functional requirements such as position, velocity, and acceleration of a mass or position, energy and power associated with a work function. Further, recent developments in dynamic balancing [27] make it fully possible to place complex linkages in a high speed system and still ensure smooth operation. Springs, dashpots, and masses can all be added to

the system to enhance this smoothness. Linkages will continue to consume more space than other systems and not all their properties can be controlled by direct synthesis techniques. Hence, the role of optimization will always prove dominant for critical design applications and therefore, a full computational capability based on synthesis to reduce the number of free design parameters will be required.

A. Dynamic Specs Synthesis of a Stephenson 3 Mechanism as a Transfer Device Between Linear and Rotary Conveyors

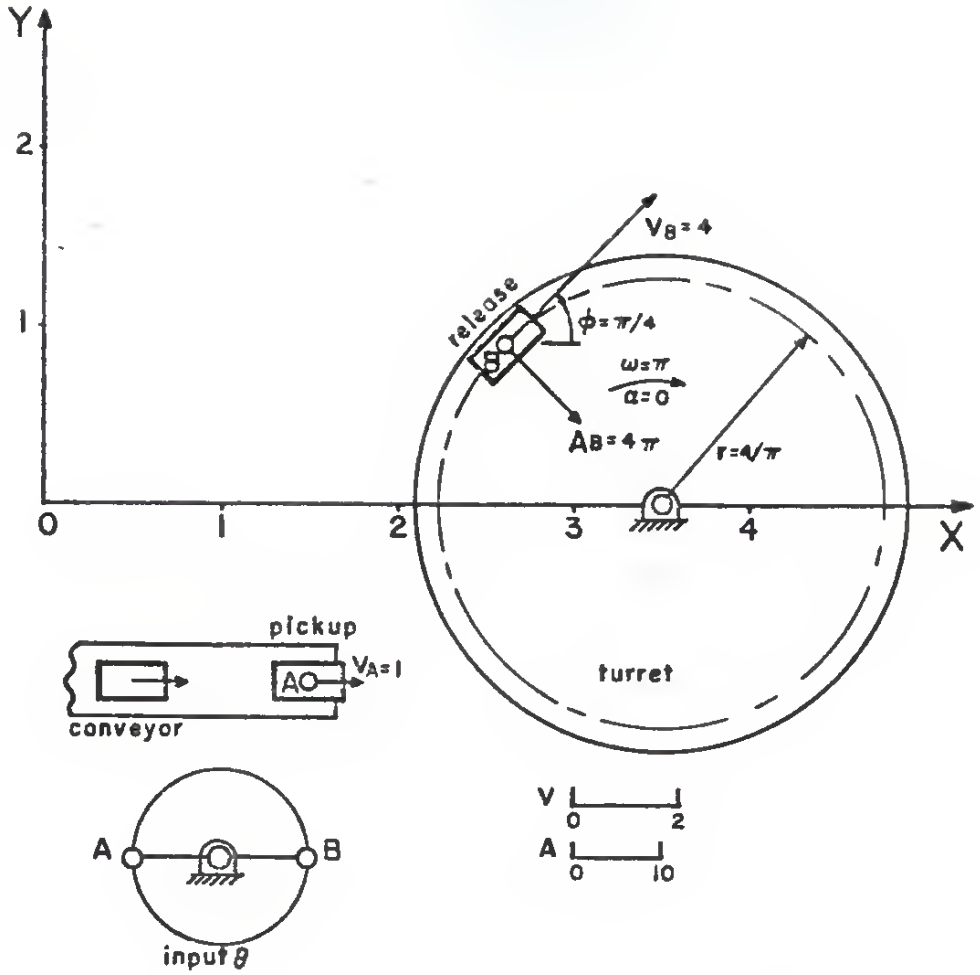
A common problem in materials handling is that of transferring an object from a continuously moving linear conveyor belt to a continuously rotating turret. If the linear surface speeds of the two devices are close in magnitude, the objects frequently can be guided by fixed plows onto the turret surface. It may be possible to design a mechanism by geometric motion specifications to match the required path properties (i.e., tangent, curvature, etc.) should the plow concept prove inadequate. If, however, the linear surface speeds differ markedly, then it is unlikely that the required matching of velocity, acceleration, jerk, etc., would result and dynamic shock will be experienced by the object during the transfer operation. In some cases where the mass and rotational inertia of the object is low or if the object or machine function is largely insensitive to shock, this mismatch of dynamic specs may not be deleterious. On the other hand, matching of the dynamic specs could improve system precision of operation or allow a significant increase in production rates.

In this section, a mechanism will be designed using dynamic specs synthesis which will match the dynamic specs of the object to the

dynamic conditions it would experience on the turret surface. Also, the angular and linear velocity of the object at pick-up will match the velocity condition of the belt. Finally, these output dynamic specs will be coordinated with the available input from the power shaft of the system.

The objects are spaced 1 foot apart on the conveyor belt moving with constant velocity of 1 foot/sec with an orientation of 0° with the horizontal and zero angular velocity. The turret rotates at a constant speed of 0.5 revolutions/sec with an effective operating radius of $4/\pi$ feet. The location of the turret gripper fingers at transfer is 135° from the X axis. At this location on the turret, the object must undergo a velocity of 4 ft/sec with an orientation of 45° with the X axis, an angular velocity matching that of the turret or 0.5 revolutions/sec (π rad/sec), and an acceleration of 4π ft/sec^2 directed toward the center of the turret. The input shaft has a constant angular velocity of 2π rad/sec. An object must be transferred during each 1-sec interval, while the actual transfer process is to consume 1/2 of this interval or 0.5 sec leaving 0.5 sec for the mechanism to return to the beginning of the cycle. Two stages are provided on the turret 180° apart to allow two objects to be placed on the turret. After the necessary turret operation on the object, it may be removed by an identical mechanism on the opposite side. The component dynamic specifications for this problem are illustrated in Fig. (4-1).

The stated synthesis problem is now presented in terms of the three design steps outlined in Chapter II.



A - PICKUP

$X = 1.5$ ft
$Y = -1.0$ ft
$\phi = 0$ rad
$\theta = \pi$ rad
$\dot{X} = 1.0$ ft/sec
$\dot{Y} = 0$ ft/sec
$\dot{\phi} = 0$ rad/sec
$\dot{\theta} = 2\pi$ rad/sec

B - RELEASE

$X = 2.6$ ft
$Y = 0.9$ ft
$\phi = \pi/4$ rad
$\theta = 0$ rad
$\dot{X} = 2.8284$ ft/sec
$\dot{Y} = 2.8284$ ft/sec
$\dot{\phi} = \pi$ rad/sec
$\dot{\theta} = 2\pi$ rad/sec
$\ddot{X} = 8.8858$ ft/sec^2
$\ddot{Y} = -8.8858$ ft/sec^2
$\ddot{\phi} = 0$ rad/sec^2
$\ddot{\theta} = 0$ rad/sec^2

Figure 4-1 Problem Specifications

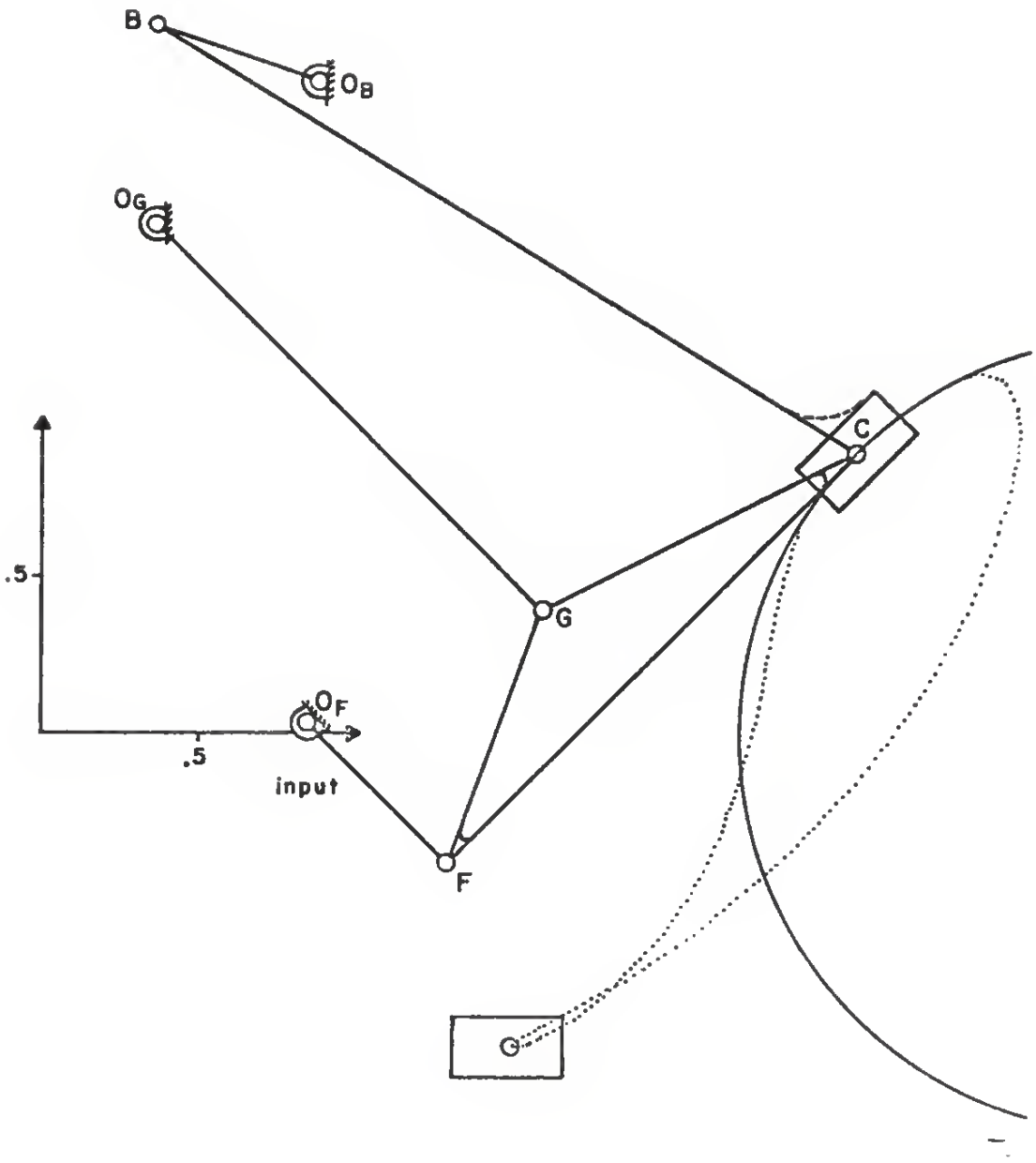


Figure 4-2 Transfer Mechanism - Drop-Off Position

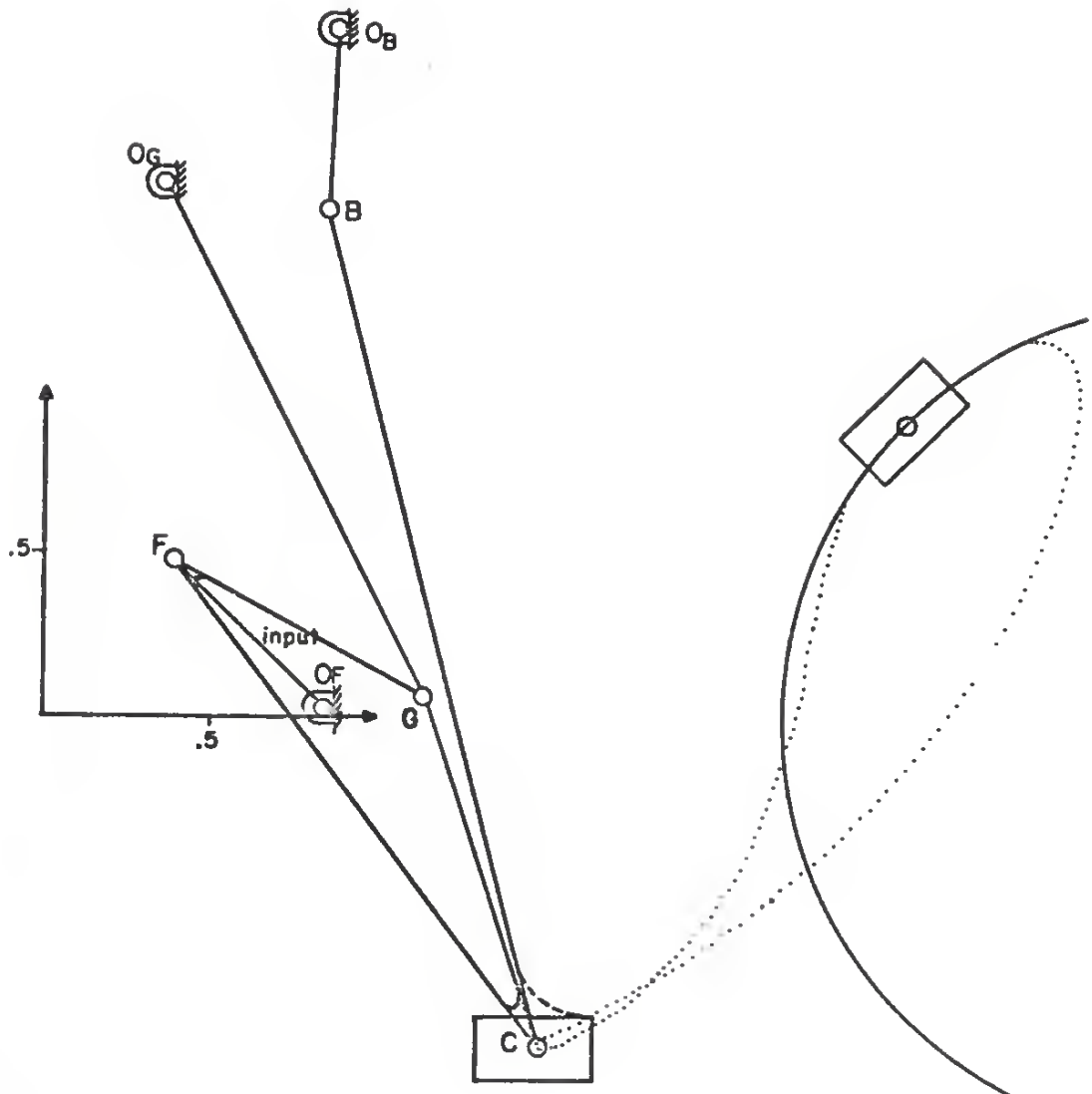


Figure 4-3 Transfer Mechanism - Pick-Up Position

Step 1. Prescribe the dynamic specs as given in Fig. (4-1):

ℓ	j	k	(k) x_j	(k) y_j	(k) ϕ_j
0	0	0	2.6 ft	0.9 ft	45 deg
1	0	1	2.8284 ft/sec	2.8284 ft/sec	π rad/sec
2	0	2	8.8858 ft/sec^2	-8.8858 ft/sec^2	0 rad/sec^2
3	1	0	1.5 ft	- 1 ft	0 deg
4	1	1	1 ft/sec	0 ft/sec	0 rad/sec

Using the formulation of Chapter II, consider Case 3 (PPP-PP) and determine by the dynamic curvature transform a single constraint crank BO_b to satisfy these specifications. Fig. (4-2) shows the final choice for BO_b in the first position of the system. Note that the first finite position (j=0) is chosen at the drop-off location to agree with the computer program MECSYN formulation which requires that any finite position for which $k \geq 2$ be specified first. This can always be done without loss of generality (even for such cases as PPP-P-P) since the sequence in which the mechanism passes through finitely separated positions (FSP) is independent of the labeling required for the analytical formulation. The numerical coordinates of the selected dyad CBO_b are:

C (2.6, 0.9), B (0.361, 2.252), O_b (0.873, 2.079).

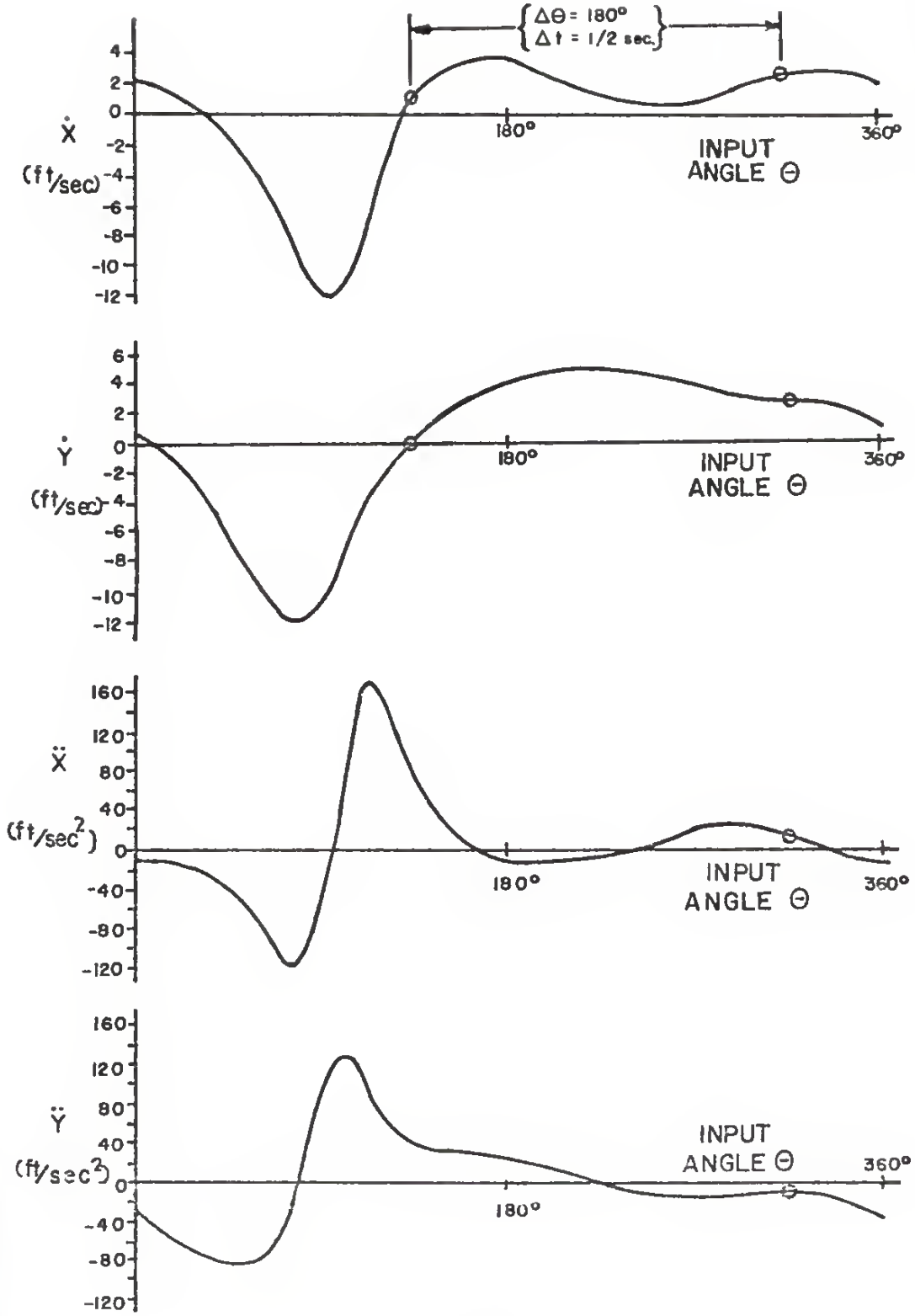


Figure 4-4 Transfer Mechanism - Path Properties

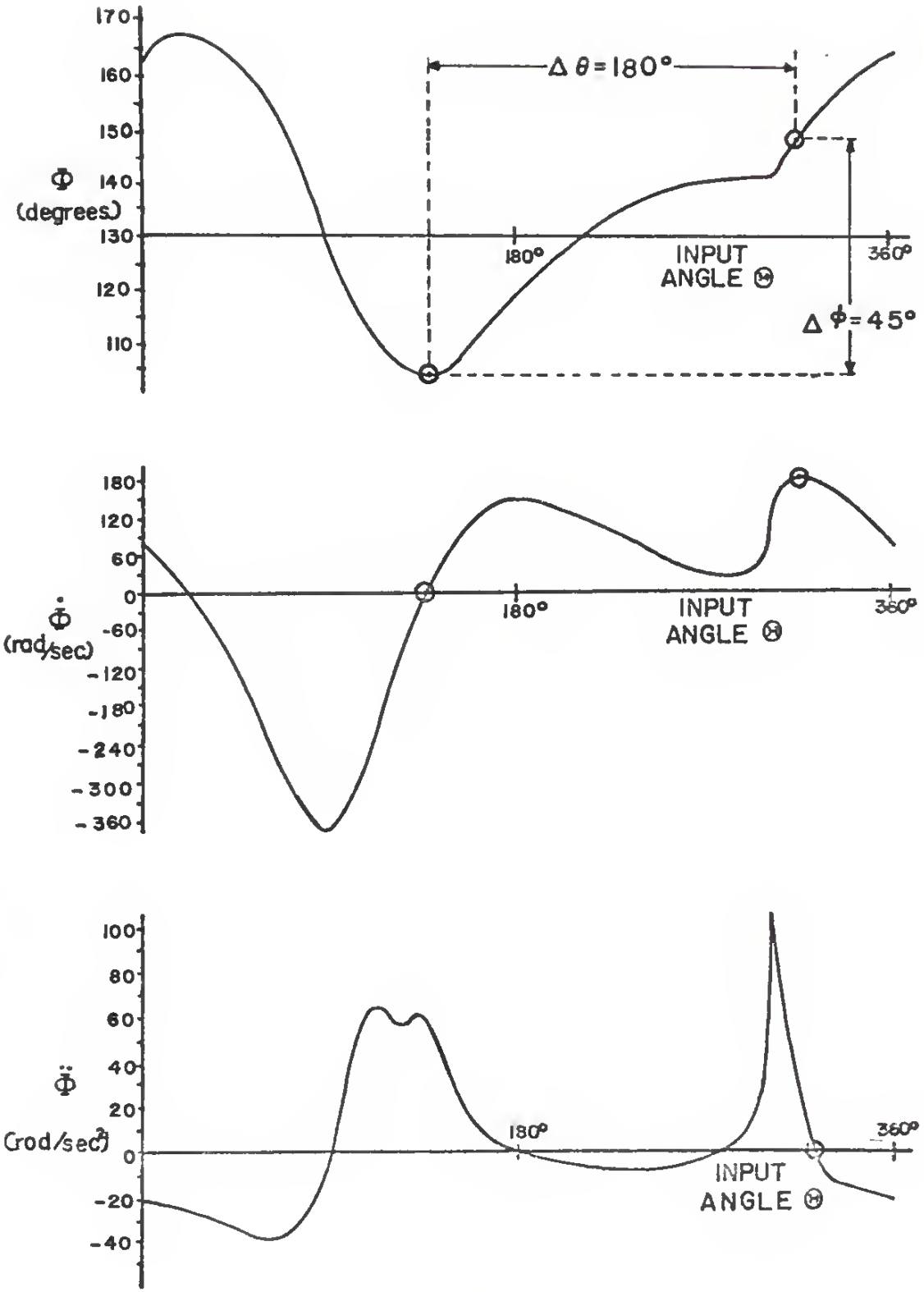


Figure 4-5 Transfer Mechanism - Angular Properties

Step 2. Prescribe the specs of point C and the input specs of the power shaft for the dynamic specs of a moving plane:

ℓ	j	k	$x_j^{(k)}$	$y_j^{(k)}$	$\theta_j^{(k)}$
0	0	0	2.6 ft	0.9 ft	0 deg
1	0	1	2.8284 ft/sec	2.8284 ft/sec	2π rad/sec
2	0	2	8.8858 ft/sec^2	-8.8858 ft/sec^2	0 rad/sec^2
3	1	0	1.5 ft	-1.0 ft	180 deg
4	1	1	1.0 ft/sec	0 ft/sec	2π rad/sec

Using MECSYN find two constraint links forming a four-bar linkage that satisfy these dynamic specifications. Path cognates of this four-bar linkage will have one crank (FO_f) which undergoes the desired input specs $\theta_j^{(k)}$ while point C experiences the specified specs $\{x_j^{(k)}, y_j^{(k)}\}$. The final choice of the four-bar cognate to satisfy this step is shown in Fig. (4-2) as GO_g, FO_f, C with the numerical coordinates:

G (1.591, 0.395) O_g (0.356, 1.619)

F (1.299, -0.414), O_f (0.846, 0.033).

Step 3. The resulting link systems of Steps 1 and 2 have in common the specs $\{x_j^{(k)}(t), y_j^{(k)}(t)\}$ at point C. Joining the dyad to the cognate four-bar gives the desired Stephenson 3. The motion of the mechanism in its two distinct design positions $j = 0,1$ is illustrated in Figs. (4-2) and (4-3) with accompanying point path for C. Fig. (4-4) shows the x,y components of velocity and acceleration for point C over the whole cycle. Figure (4-5) illustrates the angular position, angular

velocity, and angular acceleration of link CB as $\phi_j^{(k)}$. All are plotted versus the input angle θ. All prescribed data are indicated by circles on these graphs. The conditions as specified were confirmed by two independent analysis procedures to further validate the results. Note the graphs do indicate relatively high local accelerations outside the control of the synthesis process. As mentioned in the introduction, such phenomena can be minimized only by careful optimization.

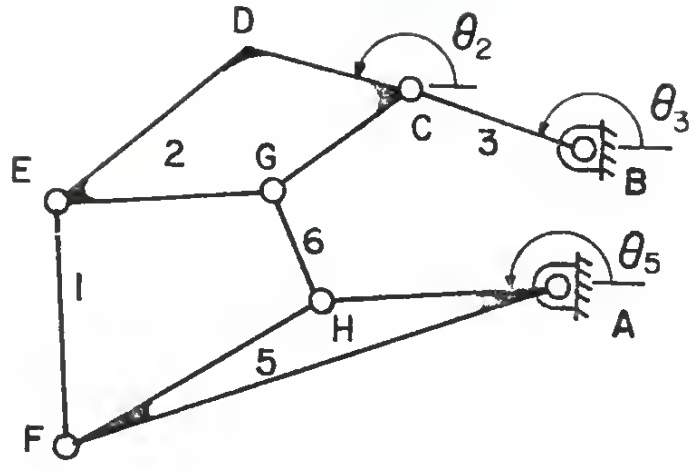
B. Dynamic Specs Synthesis of a Stephenson 2 Mechanism as a Transfer Device Between Linear and Rotary Conveyors

Using the same problem description and data given for the Stephenson 3 problem, a Stephenson 2 mechanism will be synthesized to illustrate the application of the four tools of synthesis in a modular fashion to a more difficult mechanism synthesis. The first objective in the synthesis procedure is to convert the desired motion specifications into numerical specifications suitable for use in the dynamic curvature transform central to MECSYN. The specifications for the Stephenson 3

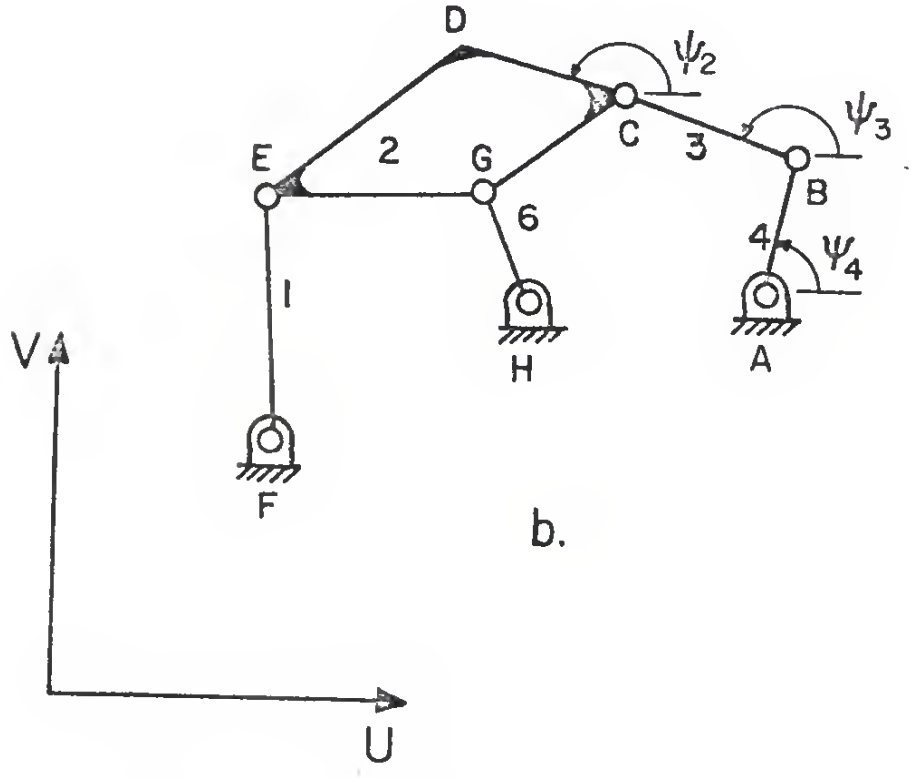
$$\left\{ x_{Cj}^{(k)}(t), \quad y_{Cj}^{(k)}(t), \quad \phi_j^{(k)}(t), \quad \theta_j^{(k)}(t) \right\}$$

are now described by a set of symbols corresponding to those shown in Fig. (4-6a) where point D replaces C as the path generator with the specifications

$$\left\{ x_{Dj}^{(k)}(t), \quad y_{Dj}^{(k)}(t), \quad \theta_{5j}^{(k)}(t), \quad \theta_{2j}^{(k)}(t) \right\} .$$



a.



b.

Figure 4-6 Two Inversions of the Stephenson Six-Link Chain

In addition the location (U_A, V_A) of pivot A can be prescribed in the reference $\Sigma(U,V)$.

Step 1. The first requirement is to determine the link configuration BCD as a segment of links 2 and 3 of Stephenson 2. This link configuration is identical to $O_b BC$ in the Stephenson 3 so that the means to satisfy this step is the same as Step 1 for the Stephenson 3. It is not necessary that the same link solution from the dynamic curvature transform be used as before. The same solution is used here, however, to illustrate the similarity between the two mechanism problems. The numerical coordinates of this solution are given as:

A (1.25, 1.25), D (2.6, 0.9)

C (0.361, 2.252), B (0.873, 2.079) .

Step 2. In this part of the synthesis, the Stephenson 2 is inverted by releasing link 4 and fixing link 5 giving a Stephenson 3 as illustrated in Fig. (4-6b). It follows from the analytical rules of inversion that the specs of point C and the angular specs of link 2 can be obtained. This means that the dynamic specs of link 2 of the Stephenson 3 can be prescribed. The chosen point A along with the synthesized points B and C from Step 1 form the dyad structure ABC. This dyad can be analyzed to provide the specs of point C after the inverted angular specs $\psi_{4j}^{(k)}$, $\psi_{3j}^{(k)}$ are determined. MECSYN provides this dyad analysis and angular inversion for dynamic synthesis according to case number of DMSP. The angular inversion $\{\theta_{mj} \rightarrow \psi_{mj}\}$ between the

Stephenson 2 and the Stephenson 3 for link m for position j is given by the formula

$$\psi_{mj} = \theta_{mj} - \theta_{5j} + \theta_{50}$$

where θ_{50} is the initial value of θ_5 in the invariant reference system $\Sigma(U,V)$. The derivative of this displacement relation gives the angular inversion of specs $\{ \overset{(k)}{\theta}_{mj} \rightarrow \overset{(k)}{\psi}_{mj} \}$ as

$$\overset{(k)}{\psi}_{mj} = \overset{(k)}{\theta}_{mj} - \overset{(k)}{\theta}_{5j} + \overset{(k)}{\theta}_{50}, \qquad m = 2,3,4 .$$

Note that all angles are measured from the same (U,V) reference system for both inversions. The inverse of the above process $\{ \overset{(k)}{\psi}_{mj} \rightarrow \overset{(k)}{\theta}_{mj} \}$ is

$$\overset{(k)}{\theta}_{mj} = \overset{(k)}{\psi}_{mj} - \overset{(k)}{\psi}_{4j} + \overset{(k)}{\psi}_{40}$$

since link 4 is fixed in the Stephenson 2. Now we may obtain by direct analysis the specs of link C for the Stephenson 3 as

$$\overset{(k)}{x}_{Dj} = \frac{d^k}{dt^k} \{ \overline{AB} \cos \psi_4 + \overline{BC} \cos \psi_3 + U_A \} \Big|_{t=t_j}$$

$$\overset{(k)}{y}_{Dj} = \frac{d^k}{dt^k} \{ \overline{AB} \sin \psi_4 + \overline{BC} \sin \psi_3 + V_A \} \Big|_{t=t_j} .$$

This completes the desired dynamic specs

$\{ \overset{(k)}{x}_{Dj}, \overset{(k)}{y}_{Dj}, \overset{(k)}{\psi}_{2j} \}$ for the inverted properties of link 2 in Fig. (4-6b).

Step 3. Using the same numerical specifications from Section 1, the initial input data are

ℓ	j	k	$\theta_{5j}^{(k)}(t)$	$\theta_{2j}^{(k)}(t)$
0	0	0	0 deg	45 deg
1	0	1	2π rad/sec	π rad/sec
2	0	2	0 rad/sec^2	0 rad/sec^2
3	1	0	270 deg	0 deg
4	1	1	2π rad/sec	0 rad/sec

with the exception that θ_{51} was changed from 180° to 270° to achieve a somewhat better solution. By the formulation of this step, the dynamic specifications for link 2 in the Stephenson 3 are:

ℓ	j	k	$x_{Dj}^{(k)}(t)$	$y_{Dj}^{(k)}(t)$	$\psi_{2j}^{(k)}(t)$
0	0	0	0.3612 ft	2.2521 ft	45 deg
1	0	1	4.8770 ft/sec	1.3796 ft/sec	$-\pi$ rad/sec
2	0	2	13.2291 ft/sec^2	-43.9557 ft/sec^2	0 rad/sec^2
3	1	0	0.9608 ft	0.8730 ft	-270 deg
4	1	1	-2.3689 ft/sec	2.8169 ft/sec	-2π rad/sec .

With these dynamic specifications, the dynamic curvature transform of MECSYN can be used to find the constraint links EF and GH to constrain link 2 relative to link 5 in Fig. (4-6b). The numerical coordinates for the chosen constraints are

E (0.940, 2.313), F (0.899, 1.724)

G (2.716, 2.475), H (1.441, 1.293) .

Step 4. The inversion to the Stephenson 2 by fixing link 4 from the Stephenson 3 has all the desired dynamic specifications for link 2 with point D the path generator and $\theta_5(t)$ the desired input specs. Figure (7) illustrates the resulting Stephenson 2 mechanism. The dashed path is that for point C of the corresponding inversion to the Stephenson 3 mechanism. Although the resulting mechanism was satisfying for many reasons, the input angle θ_5 could not be increased monotonically even though it could be rotated completely in the same closure for the mechanism. Therefore, the Stephenson 3 mechanism of Section 1 synthesized for the same specifications is a better mechanism for application to the stated transfer problem. Note that varying any of the motion specifications a small amount can dramatically affect the suitability of the resulting mechanisms. Also, the coordinates U_A, V_A can be changed arbitrarily without changing the basic transfer problem. The coordinates of point A were varied many times resulting in numerous distinct Stephenson 2 mechanisms. The final result satisfies more of the necessary physical design criteria than the others. The first 3 steps of this section are illustrated in Table 4-1 consisting of output from MECSYN.

As indicated in Chapter II, a Stephenson 1 mechanism can also be synthesized for the same specifications, being more difficult to design than a Stephenson 3, yet less difficult to design than a Stephenson 2. The author concludes that for demanding specifications as in these examples, a Stephenson 3 generally provides better practical results for the least effort.

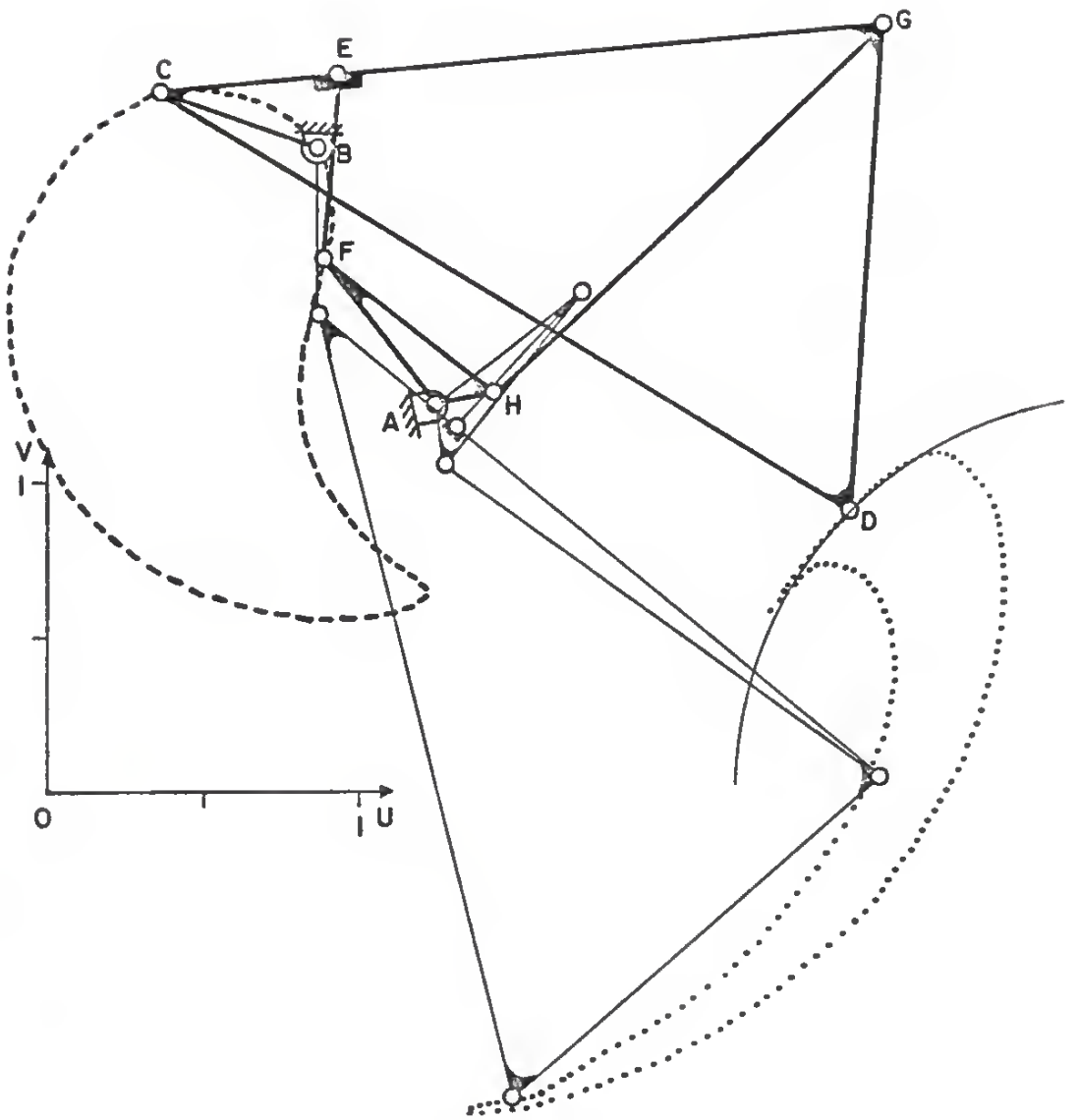


Figure 4-7 Transfer Mechanism - Stephenson 2

Table 4-1 Stephenson 2 Synthesis

NECSYN

SYNTHESIS OF MECHANISMS
FIVE MULTIPLY SEPARATED POSITIONS
IN COPLANAR MOTION - CASE 3

OPTIONS

STRAIGHT LINE MOTION	0.0
TRACE	0.0
COGNATE MECHANISMS	0.0
ANALYSIS	1.0

SPECIFICATIONS

ALPHA(0.0)	=	2.0000
BETA(0.0)	=	0.9000
GAMMA(0.0)	=	A5.0000
ALPHA(0.1)	=	2.8234
BETA(0.1)	=	2.8294
GAMMA(0.1)	=	3.1A16
ALPHA(0.2)	=	0.8856
BETA(0.2)	=	-0.8858
GAMMA(0.2)	=	0.0
ALPHA(1.0)	=	1.5000
BETA(1.0)	=	-1.0000
GAMMA(1.0)	=	0.0
ALPHA(1.1)	=	1.0000
BETA(1.1)	=	0.0
GAMMA(1.1)	=	0.0

COORDINATES OF THE CONSTRAINT LINKS

	POINTS				CENTERS		
X =	1.50079	Y =	).AT958	R =	1.13255	9 =	0.08577
X =	0.36117	Y =	2.25209	X =	0.87296	Y =	2.07938

COUPLER POINT
1 2.600. 0.000)

ORIGINAL MECHANISM			FIRST COGNATE			SECOND COGNATE		
A1	1.501.	1.4801	O1	2.232.	0.308)	F1	3.112.	0.7271
OA(	1.131.	0.9491	OD(	1.133.	C.8961	OF(	0.873.	2.0791
B(	0.3A1.	2.2521	E(	1.961.	-0.2A3)	G(	2.12A.	1.004)
OB(	0.873.	2.0791	OE(	1.48A.	-0.159)	OG(	1.A84.	-0.189)

CRANK-ROCKER		CRANK-ROCKER		DOUBLE-ROCKER	
LINK LENGTH RATIO	2.55	LINK LENGTH RATIO	2.55	LINK LENGTH RATIO	2.53
MODIFIED L.L.R.	3.57	MODIFIED L.L.R.	2.55	MODIFIED L.L.R.	5.09

INPUT PARAMETER CONVERSION FOR
SIX-LINK AND GEARED FIVE-LINK MECHANISMS
BASED ON STEPHENSON-2 SIX-LINK MECHANISM

CASE NUMBER 3

POINT A		POINT B		POINT C	
1 1.2500.	1.2500)	(0.9730.	2.0793)	(0.3812.	2.2521)

LINK 3		LINK 4	
LENGTH	ANGLE	LENGTH	ANGLE
0.5402	101.3A90	0.9110	114.4407

POS.	DER.	STEPHENSON-2 THETA 2	STEPHENSON-2 THETA 3	STEPHENSON-2 THETA 4	STEPHENSON-3 PSI 3	STEPHENSON-3 PSI A
0	0	A5.0000	0.0	101.3491	101.3A01	114.4407
0	1	3.1410	8.2832	8.2141	1.9329	-8.2832
0	2	0.0	0.0	20.8503	20.8503	0.0
1	0	0.0	270.0000	270.0000	0.0	-155.5533
1	1	0.0	8.2832	1.8812	-A.4320	-8.2832

POS.	DER.	XC	YC	PSI 2
0	0	C.3812	2.2521	45.0000
0	1	4.8770	).3796	-3.1418
0	2	13.2291	-43.9557	0.0
1	0	0.9808	0.8730	-270.0000
1	1	-2.3889	2.0189	-8.2832

COORDINATES OF THE CONSTRAINT LINKS

	POINTS				CENTERS		
X =	0.03656	Y =	2.3134R	R =	0.68869	Y =	1.72386
X =	1.15770	Y =	0.7A578	R =	1.94917	Y =	0.84777
X =	2.71843	Y =	2.47519	X =	1.A4057	Y =	1.29298
R =	-21.485A8	Y =	-11.59263	X =	1.35043	Y =	1.00340

ORIGINAL MECHANISM			FIRST COGNATE			SECOND COGNATE		
A(	C.94C.	2.313)	O(	0.119.	1.582)	F(	-0.913.	1.0731
OA(	0.889.	1.724)	OD(	0.889.	1.724)	OF(	1.441.	1.293)
B(	2.718.	2.474)	E(	0.237.	1.47C)	G(	0.7T1.	2.6441
OB(	1.441.	1.253)	OE(	0.707.	1.882)	OG(	0.707.	1.882)

CRANK-ROCKER		DOUBLE-ROCKER		CRANK-ROCKER	
LINK LENGTH RATIO	3.01	LINK LENGTH RATIO	3.01	LINK LENGTH RATIO	3.01
MODIFIED L.L.R.	3.01	MODIFIED L.L.R.	3.57	MODIFIED L.L.R.	3.01

C. Geometric Specs Synthesis of a Stephenson 3 Mechanism for Cutting Large, Low Pitch Gear Teeth

This design illustrates the special case of geometric synthesis for a Stephenson 3 six-link mechanism as defined in Chapter II. The specifications for the Stephenson 3 for dynamic synthesis are

$$\left\{ {}^{(k)}x_j(t), \quad {}^{(k)}y_j(t), \quad {}^{(k)}\phi_j(t), \quad {}^{(k)}\psi_j(t) \right\}.$$

It was shown in Chapter II that the dynamic specs formulation contains as a subset the MSP geometric synthesis for a link moving with general coplanar motion relative to input displacement of a crank. The subset is obtained by replacing t with the input ψ giving the specification for geometric synthesis as

$$\left\{ {}^{(k)}x_j(\psi), \quad {}^{(k)}y_j(\psi), \quad {}^{(k)}\phi_j(\psi), \quad {}^{(k)}\psi_j \right\}$$

where

$$\dot{\psi} = 1, \quad \ddot{\psi} = \dddot{\psi} = \ddddot{\psi} = 0 .$$

The goal here is to cut by mechanically controlled motion the teeth of a very large, low pitch gear. The gear is an 8-tooth, 3/8 pitch, 21.333 inch pitch diameter, 6 inch face width involute spur gear. The following quote comes from Ref. [28]:

> It is still very difficult to cut a gear tooth coarser than 3/4 pitch. Lufkin Foundry and Machine Company have recently made some very well-formed gear teeth twice this big at 3/8 pitch.

The gear teeth can be cut either by oxygen or bandsaw cutting. With oxygen cutting, it may be more efficient to use a shaped flame and to maintain a given flame orientation to the tooth surface. For bandsaw cutting, it is essential that the saw blade be tangent to the tooth

surface. In either case, it is desirable to have constant cutting speed while the driving shaft angular velocity is constant to produce a uniform cutting operation. Also, a high degree of accuracy in the cutting path, tangent angle and constant speed condition is considered important.

A Stephenson 3 mechanism will be synthesized to satisfy the posed problem. Higher order motion properties will be derived from the definition of the input and the involute curve and corresponding involute angle. The MSP specifications will be obtained for case PPP-PP to achieve close matching of the involute geometry. The cutting point will be point C in the synthesis formulation while the angle ϕ of the corresponding coupler BC will generate the involute angle to maintain the tangency requirement for proper cutting.

The driving crank angle ψ is defined by the following conditions:

1. Cutting speed is constant when input speed is constant

$$\frac{dR}{dt} = K\ r\ \frac{d\phi}{dt}$$

where

r - pitch radius

K - input scale factor

R - reference vector magnitude for point on the involute

ϕ - involute angle.

2. Input conditions are initially zero, $\psi_0 = 0$ at $\phi = 0$.

From condition 1 above

$$\int_{R_0}^{R} dR = K\ r \int_{\psi_0}^{\psi} d\psi$$

with $R(0) = R_0$. This shows that

$$R - R_0 = K\, r\, \psi \ .$$

3. The involute curve is described parametrically in terms of the involute angle ϕ by

$$x = r(\cos\phi + \phi \sin\phi)$$

$$y = r(\sin\phi - \phi \cos\phi)$$

such that

$$R = \sqrt{x^2+y^2} = r\sqrt{1+\phi^2}, \quad R(0) = R_0 = r.$$

Hence, the input ψ is defined as

$$\psi = \frac{1}{Kr}(R-R_0) = \frac{1}{K}(\sqrt{1+\phi^2} - 1) \ .$$

The first and second derivatives of the functions $\phi(\psi)$, $x(\psi)$, $y(\psi)$ are required for the geometric synthesis of case PPP-PP. These data are given by

$$\phi(\psi) = +\sqrt{K^2\psi^2 + 2K\psi} = g(\psi)$$

$$x(\psi) = r(\cos g(\psi) + g(\psi) \sin g(\psi))$$

$$y(\psi) = r(\sin g(\psi) - g(\psi) \cos g(\psi))$$

$$\frac{d\phi}{d\psi} = \phi'(\psi) = \frac{K^2\psi+K}{g(\psi)}$$

$$\frac{dx}{d\psi} = x'(\psi) = rK(K\psi+1) \cos g(\psi), \quad \frac{dy}{d\psi} = y'(\psi) = rK(K\psi+1) \sin g(\psi)$$

$$\frac{d^2\phi}{d\psi^2} = \phi''(\psi) = \frac{-K^2}{[g(\psi)]^3}$$

$$\frac{d^2x}{d\psi^2} = x''(\psi) = r\{K^2 \cos g(\psi) - [K^2 g(\psi) + \frac{K^2}{g(\psi)}] \sin g(\psi)\}$$

$$\frac{d^2y}{d\psi^2} = y''(\psi) = r\{K \sin g(\psi) + [K^2 g(\psi) + \frac{K}{g(\psi)}] \cos g(\psi)\} \ .$$

The description of the coarse pitch involute spur gear, 20° stub tooth by AGMA Standards is:

$$P_d = \frac{8}{21.333} = 0.375 = 3/8$$

$$\text{Addendum} = \frac{0.8}{P_d} = 2.133"$$

$$\text{Minimum Dedendum} = \frac{1}{P_d} = 2.667"$$

$$\text{Working Depth} = \frac{1.6}{P_d} = 4.267"$$

$$\text{Circular Tooth Thickness} = 4.189"$$

$$r_{base} = (r_{pitch}) \cos 20° = 10.024"$$

ϕ_p (at pitch circle) = 20.849°

ϕ_a (at addendum circle) = 45.497° .

To obtain the scale factor K, choose the first finite position j = 0 (with the PPP) close to the pitch point, say $\phi \approx 21°$ and the second finite position j = 1 (with the PP) close to the addendum circle, say $\phi \approx 40°$ making $\Delta\phi_{0\to1} = 19°$. Also, choose the crank rotation angle for these positions to be $\Delta\psi_{0\to1} = 75°$. Then it follows that

$$\psi_0 = \frac{1}{K}(3.7272°), \quad \psi_1 = \frac{1}{K}(12.5813°)$$

for $\phi_0 = 21°$ and $\phi_1 = 40°$. Noting that $\Delta\psi_{0\to1} = 75°$ the value of K becomes K = 0.1181. The crank positions from the above expressions using this value of K are

$$\psi_0 = 31.572°, \qquad \psi_1 = 106.572° .$$

The synthesis of the mechanism follows the 3 steps outlined in Chapter II. These are:

Step 1. For the plane BC which controls the cutting device the motion specifications are

ℓ	j	k	$x_j^{(k)}(\psi)$	$y_j^{(k)}(\psi)$	$\phi_j^{(k)}(\psi)$
0	0	0	10.6748	0.1623	21.0°
1	0	1	1.1766	0.4517	0.3431
2	0	2	-0.0245	0.4513	-0.2831
3	1	0	12.1771	1.0825	40.0°
4	1	1	1.1056	0.9277	0.2062

Using the dynamic curvature transform in MECSYN with the replacement ($t \sim \psi$), synthesize one constraint link BO_b to control the motion of coupler BC to form the dyad CBO_b. The numerical coordinates for the points in this dyad are

C (10.675, 0.162), B (-16.237, -5.254), O_b (2.444, 1.204)

Step 2. This step is the solution of an ordinary geometric synthesis problem. Specify the same specs $(x_j^{(k)}(\psi), y_j^{(k)}(\psi))$ as before for point C. The corresponding values for the input specs $\psi_j^{(k)}$ are 31.572°, 1, 0, 106.572°, 1 for sequential positions ℓ. In the dynamic curvature transform use the replacement ($t \sim \psi$) to synthesize by MECSYN two constraints on the coupler link of a four-bar linkage. The cognate DO_dEO_e with coupler point C has the geometric specs $(x_j^{(k)}(\psi), y_j^{(k)}(\psi))$ at point C coordinated with the input $\psi_j^{(k)}$. The numerical coordinates of the significant points of this cognate are:

D (7.977, 1.731), O_d (9.208, 1.929)

E (10.144, 1.017), O_e (9.552, 4.168)

where DO_d is the input crank.

Step 3. The dyad CBO_b of Step 1 and the four-bar linkage DO_dEO_e have the state at point C in common. Joining these systems at C forms the desired Stephenson 3 gear cutter mechanism.

Figure (4-8) illustrates the gear cutter mechanism and the gear geometry. The curves in Fig. (4-9) illustrate the quality of the resulting approximation for the x,y motion of the cutter along the involute tooth and the corresponding error curves. The crank rotation angle ψ is used as the abscissa with initial ψ corresponding to $\phi = 0°$ at the base circle and final ψ corresponding to $\phi \simeq 46°$ on the addendum circle. These curves show that the involute is generated with very high accuracy except at the ends of the motion. However, the gear tooth will be undercut at both ends consistent with the direction of the error. If the desired undercut is exceeded, respacing the precision points may produce a better solution.

Figure (4-10) shows the error curve for the coupler angle compared with the true involute angle which is surprisingly small. It could be further improved by moving the $j = 0$ location to the left. The other curve in Fig. (4-10) shows the variation in the velocity ratio $\Delta s/\Delta\psi$ which should be constant so that the involute is cut at a constant rate. The curve shows a maximum error of $\pm$ 1.5% which should prove very satisfactory for the intended purpose.

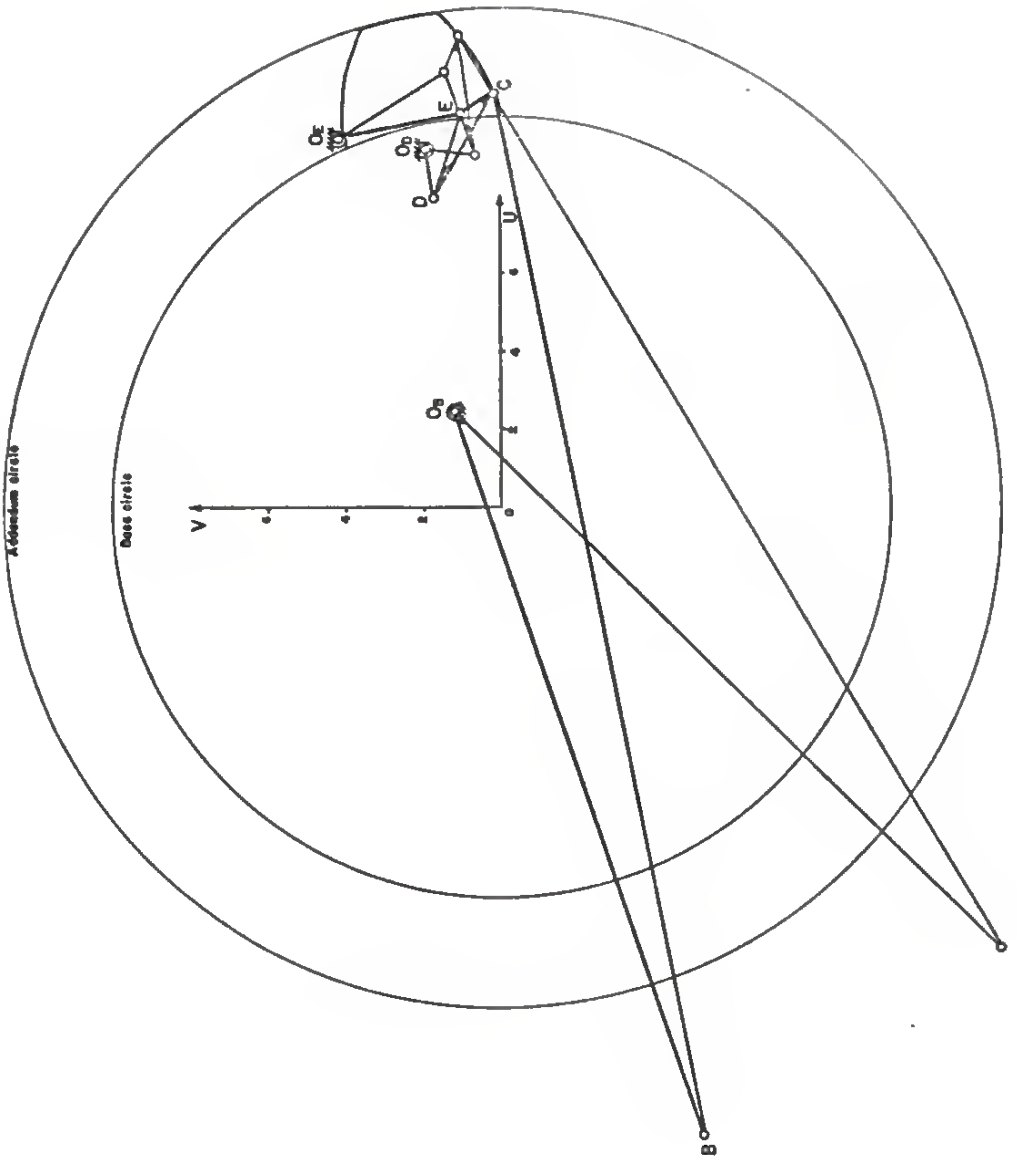


Figure 4-8 Gear Cutter Mechanism

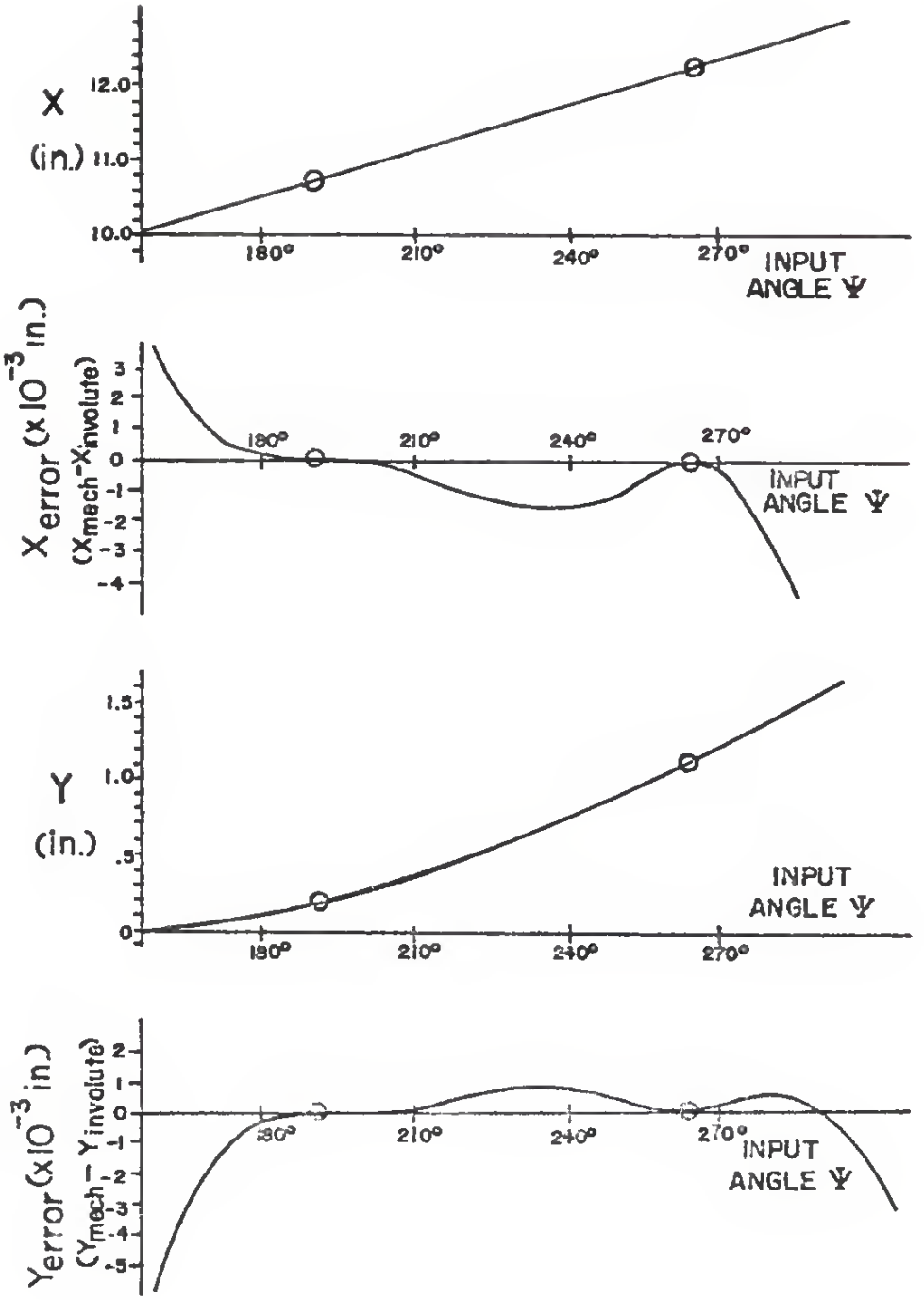


Figure 4-9 Path and Error of Gear Cutter Mechanism

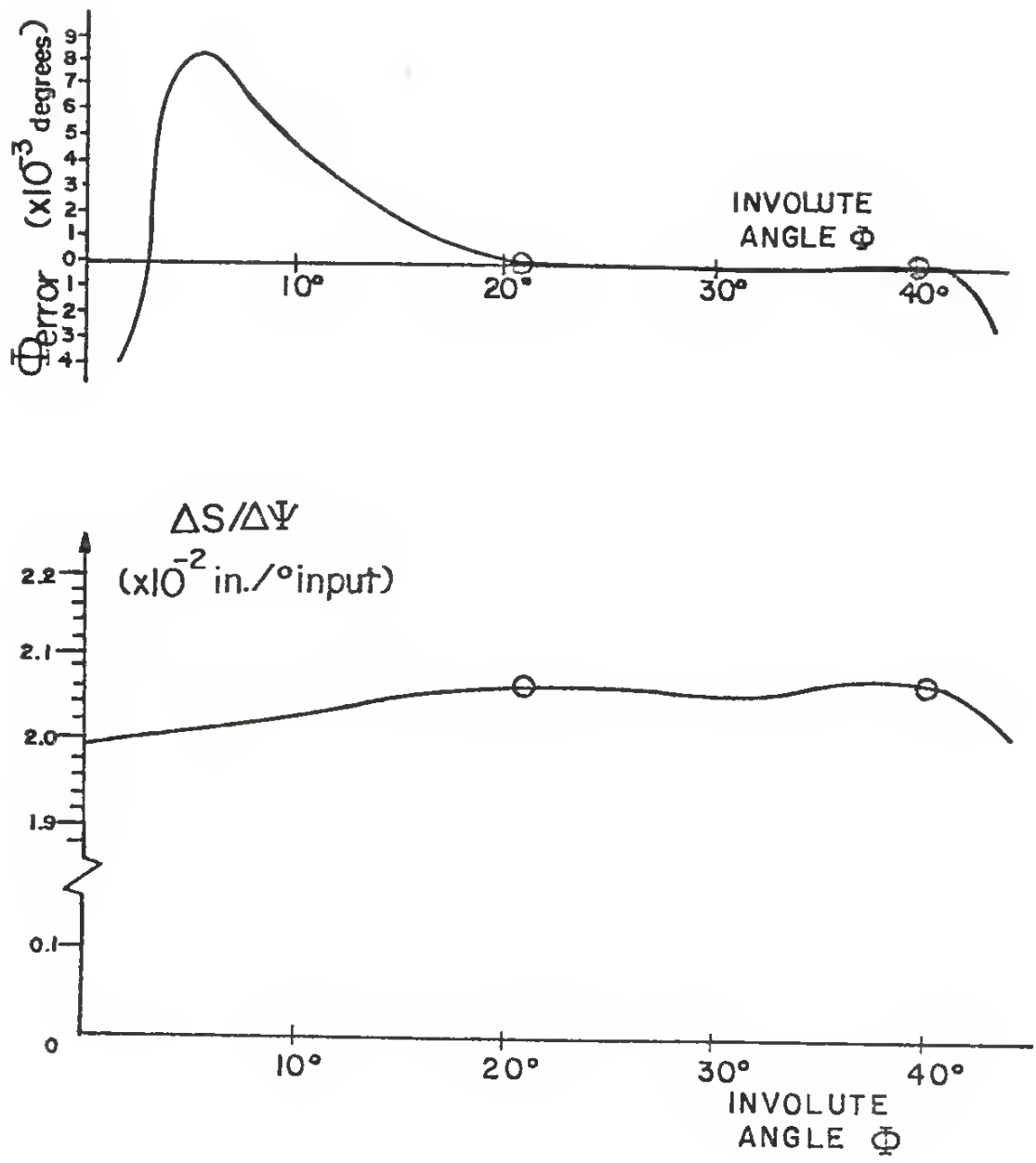


Figure 4-10 Angular Error and Cutting Rate

CHAPTER V

MECSYN

If the previous theory is to be applied to the practical design of mechanisms, an efficient, complete computer program must be available due to the lengthy calculations involved and the necessity of multiple solutions in obtaining a good design. This chapter presents a complete, closed form, noniterative computer program along with a description of input and examples of possible output, which performs all the synthesis and analysis functions previously mentioned and several others.

The program is written in Fortran IV, avoiding features not supported by ANS Fortran so that it is system independent to a large extent. Input and output unit numbers for Fortran are the variables NI and NO which are initialized in the main program to 5 and 6. The two statements initializing these unit numbers may be changed to reflect different input/output unit numbers at other installations.

The size of the program when compiled on an IBM 370/165 with Fortran H was about 72K bytes, while the size of the load module created was about 102K bytes. Because of the closed form nature of the program, the execution costs are nominal. Execution of the load module for the representative examples following the program listing was about 85 cents on the University of Florida's IBM 370.

As much as possible, the output has been left justified to reduce output time due to spacing on a time-sharing terminal. Also, the author has tried to make the input as simple as possible and uniform for all options.

In the following, the terms "four-position" and "five-position" refer to all possible combinations of finitely separated positions and time derivatives up to fourth order for those problems.

A. Options Record

The input for all problems is preceded by an options record. This record indicates which options of MECSYN will be applied to the problem data in addition to providing a title input for each problem. The format of this record follows:

Columns	Description
0-30	Up to six four-letter option words separated by any character. Blanks and unrecognized words are ignored. The option words may be in any order.
31-63	Any 32 character problem title.
66-70	START
71-75	STOP
76-80	STEP

The last three items are optional floating point variables which prescribe the sweep through the plane for solutions in a four-position problem. They represent the initial angle, final angle and increment in degrees (polar coordinate angle) of the solution sweep. If they are blank the default values are 1.° to 171.° in steps of 10.° which is

equivalent to 0°-170° in the sweep (0° ≡ 180°). Since the solutions are found for each ray through the origin, 0°-180° is sufficient to cover the whole plane. The default values give a maximum of 36 real solutions for constraint links.

There are nine option words which may be used in various combinations. No more than six need to be specified for any given problem. If none is specified the default option is a basic five-position solution. For all other options a five-position problem is solved unless FOUR, FRFV or COGO is specified. The options are:

Option Words	Option Description
COGS	With five-position solutions, the path cognates, Grashof criteria, link length ratio are calculated for all possible four-link mechanisms. The average length of the two rays from the pin joints to the coupler point is included in the modified link length ratio to give a measure of the distance from the four-link chain to the coupler point.
ANAL	When specified with COGS, this option gives a complete analysis of the mechanisms created with COGS for all the specified precision conditions. Positions and derivatives are calculated automatically for the values of j (position) and k (derivative) specified as input. This option assumes the path specifications are applied at the common coupler point of each ten-link path cognate mechanism and solves for the angular properties of all moving links in both closures as described in Chapter III. Sometimes this information is necessary to go from one modular synthesis step to another, in addition to verifying the synthesis results.
TRCE	This option traces the five-position solution and writes intermediate equations. Included are the values of the $D_{m\ell}$ coefficients used in the problem and the quartic polynomial giving the x coordinates of the circle points for five positions.

Option Words	Option Description
STLN	If the path specifications satisfy $y = ax + c$ and the path point derivatives satisfy, $y_j^{(k)}(t) / x_j^{(k)}(t) = a$, for five positions, a special algorithm must be carried out to obtain solutions. This option rearranges matrices and coefficients to obtain solutions (using the same basic equations) for straight line motion specifications. An error message notifies the user when this option is necessary.
FGEN	This option converts four-bar function generation input specifications to equivalent specifications for constraint link synthesis and calls the five-position or four- and five-position constraint link synthesis subprograms depending on which option is specified. This type of function generation does not require inversion but angular or path cognates as described in Chapter III. If COGS and ANAL are chosen as options also, the function generation specifications can be observed in the analysis of one path cognate mechanism. The function generator can be completed by angular cognates by using the cognates only option (COGO). Function generator four-bars are useful for synthesis problems for Stephenson 1 six-bars (Refs. [14,15]). If specified along with SIXL, this option overrides SIXL.
SIXL	This option performs input specification conversion similar to FGEN along with inversion to give equivalent specifications for path related synthesis problems of geared five-bars, Stephenson 2 six-bars and function generation for geared five-bars, Stephenson 1 and Stephenson 2 six-bars. Function generation for six-link mechanisms refers to angular coordination of three links for all combinations of positions and time derivatives. For path related synthesis problems a previous constraint link synthesis step must be run as described in Chapters III, IV and Refs. [14,15 19]. The five-position constraint link synthesis routine (or four- and five-position synthesis if FRFV is chosen) is called after the SIXL option

Option Words	Option Description
	is completed. This option may also be used for synthesis of multi-link mechanisms with more than six links.
FOUR	If only a four-position problem is desired, this option is chosen. The result is, theoretically, an infinity of solutions for constraint links governed by the sweep variables START, STOP and STEP. FOUR overrides all other option words except COGO. If it is desired to synthesize a six-link mechanism for only four positions, FRFV must be specified. That is, a four-position and a five-position problem. Then options such as FGEN and SIXL may be used and solutions are given for both four and five positions.
FRFV	This option may be chosen along with all other options excluding FOUR and COGO. The four-position solution is carried out first (after any options such as FGEN or SIXL) using the first four sets of specifications. The five-position solution follows using all five sets of specifications. This implies one extra set of finitely separated position specs or one extra set of velocity specs in addition to the four-position specs. If FOUR is specified also, the next problem will have one extra input data record.
COGO	If only path or angular cognates, Grashof criteria or link length ratios are desired, this option is specified. COGO overrides all other options. The input consists of the coordinates of five points for any four-link mechanism. If angular cognates of a four-link mechanism are desired, the coupler point is specified as one of the pin joints. The output will indicate that cognates, Grashof criteria and link length ratios are desired but the number of positions specified will be given as 0. This option allows path cognates to be calculated separately for selected four-position solutions.

All the routines in MECSYN give solutions for any possible case of multiply separated positions for time dependent specs. The case numbers are not specified but are determined in MECSYN from the input data.

There are seven possible cases for five-position solutions and five possible cases for four-position solutions. These cases are described on comment cards in Subroutine INPUT. However, it is not necessary to be familiar with them since they are used only for program control. The parameters j (position) and k (derivative) are specified for each set of input data.

As many problems may be run together as desired, each preceded by an options record as long as the correct number of data cards follow (3, 4, 5 or 6). For each problem, a heading will be printed with "yes" or "no" indicating which options were selected. If options are overriden, they will still be indicated with a "yes" but the computations will not be carried out. The number of specified positions will be indicated by 0, 4, 5 or 45 for both. Examples of input and corresponding output follows the MECSYN listing. Description of the data for each option will be given in the next section.

B. Input Data

All input data following the options record are accepted in ten-column fields so the same tab stops on a time-sharing terminal or program on a keypunch may be used for any problem. There are three basic types of input depending on which option is chosen. Values of the counters j for finitely separated positions and k for time derivatives begin at zero and are accepted as integers (fixed point). All other data are accepted in real (floating point) format.

Specifications are given in a chosen fixed reference system and all answers are given in the fixed reference system for the zero position of the resulting mechanism. Derivatives higher than two must be

specified for the zero position (without loss of generality) but the input records containing the counters j and k may be placed in any order. If option FRFV is chosen, however, the first four j,k records are taken for the four-position problem.

The description of input records will be given in the order of the number required (3-6) following the options record.

If the option COGO (cognates, etc., only) is chosen, three records follow the options record giving the necessary information about the four-link mechanism.

Record	Columns	Description
1	1-10	x coordinate } Fixed pivot of first constraint link
	11-20	y coordinate }
	21-30	x coordinate } Moving pivot of first constraint link
	31-40	y coordinate }
2	1-10	x coordinate } Fixed pivot of second constraint link
	11-20	y coordinate }
	21-30	x coordinate } Moving pivot of second constraint link
	31-40	y coordinate }
3	1-10	x coordinate } Coupler point for path cognates
	11-20	y coordinate }

If the option FOUR is chosen for a four-position problem only, four records follow the options record.

Record	Columns	Description
1-4	1	Counter, j, of finitely separated position
	2	Counter, k, of order of time derivative
	3-10	Blank (to allow uniform tab stops)
	11-20	x component } Position or time derivative specifi-
	21-30	y component } cations for the coupler point $\left(x_j^{(k)}(t),\ y_j^{(k)}(t) \right)$
	31-40	Position or time derivative specifications for an input, output or coupler angle $\left(\theta_j^{(k)}(t) \right)$

For all other options excluding FGEN and SIXL, five records in the same format as those above for option FOUR, follow the options record.

Record	Columns	Description
1-5	1	j Finitely separated position
	2	k Time derivative, $d^k( )/dt^k$.
	3-10	Blank
	11-20	$x_j^{(k)}(t)$
	21-30	$y_j^{(k)}(t)$
	31-40	$\theta_j^{(k)}(t)$

If FGEN or SIXL is chosen the input consists of the options record, one record giving the coordinates of three points, either chosen arbitrarily or solutions from a previous synthesis step and five j,k records giving the problem specifications.

When FGEN is chosen, the next record gives the coordinates of a fixed pivot, a moving pivot and a coupler point. These are chosen arbitrarily if only a four-bar problem is being solved. The remaining records give specifications for input and output angular properties.

Record	Columns	Description
1	1-10	x coordinate } Fixed pivot of a constraint link
	11-20	y coordinate }
	21-30	x coordinate } Moving pivot of a constraint link
	31-40	y coordinate }
	41-50	x coordinate } Coupler point
	51-60	y coordinate }
2-6	1	Counter, j, of finitely separated position
	2	Counter, k, of order of time derivative
	3-10	Blank
	11-20	Input angle positions or time derivatives, $(\theta_j^{(k)}(t))$ for the constraint link specified above
	21-30	Output angle positions or time derivatives, $(\phi_j^{(k)}(t))$

When SIXL is chosen, the next record gives the coordinates of two fixed pivots and a moving pivot as for a Stephenson 2 six-bar or a geared five-bar mechanism. If a path related synthesis problem is being solved, fixed pivot A may be chosen while fixed pivot, B, moving pivot, C, and one set of angular specifications are given by a previous synthesis step. If a six-bar function generation problem is being

solved, points A, B and C are chosen freely as well as all three sets of angular specifications.

A geared five-bar function generation problem allows choice of A, B, C and two sets of angular properties. The other set is predetermined by the gearing relationships. Refer to previous chapters for explanation of the notation used here.

Record	Columns	Description
1	1-10	x coordinate of point A
	11-20	y coordinate of point A
	21-30	x coordinate of point B
	31-40	y coordinate of point B
	41-50	x coordinate of point C
	51-60	y coordinate of point C
2-6	1	Counter, j, of finitely separated positions
	2	Counter, k, of order of time derivative
	3-10	Blank
	11-20	Position and time derivative specifications for θ_2, θ_1 or θ_6 ($\theta_{2j}^{(k)}(t)$)
	21-30	Position and time derivative specifications for θ_3 ($\theta_{3j}^{(k)}(t)$)
	31-40	Position and time derivative specifications for θ_5 ($\theta_{5j}^{(k)}(t)$)

All the above problems may be solved for the usual geometric synthesis, also. Input for all path properties will represent geometric

derivatives rather than time derivatives (e.g., $d^k x(t)/dt^k \rightarrow d^k x(\theta)/d\theta^k$). Input for all associated angular derivatives will be zero above first order and one for first order ($d^k\theta(t)/dt^k \rightarrow d^k\theta/d\theta^k$; $\dot{\theta} \rightarrow 1$; $\ddot{\theta}, \ldots, \dddot{\theta} \rightarrow 0$). For all the problems involving angles only (FGEN, SIXL), the input angle first derivative is specified as unity, while all higher derivatives are specified as zero. The other angular properties represent geometric derivatives with respect to input angle rather than time derivatives.

Note that all the foregoing synthesis problems for which time dependent specifications are made must have a specified time dependent input which can be obtained to drive the mechanism, as explained in previous chapters. For this reason, multi-link mechanisms are best suited to this type of synthesis (more free parameters). However, if these conditions cannot be met, MECSYN can still be used for the more restrictive, classical geometric synthesis problems by reducing the input as shown above.

C. Program Description

A brief description of the subroutines in MECSYN may be useful to the user. In addition to this description, there are several comment statements within the program. All real (floating point) variables within the program are of length 8 (double precision) excluding control parameters. Past usage has shown this to be necessary in many cases.

MAIN

The main program performs calls to subroutines but no computations other than rearranging matrices for the straight line problem, coordinate transformations on the constraint link solutions and determining the center points for the five-position solution.

INPUT

The input data are read, interpreted, control paramter values set and preliminary output is written. INPUT also calls subroutines according to which options were chosen.

EQUATN

The coefficients of the constraint equations, $D_{m\ell}$, are computed for four- or five-position synthesis. The coefficients are computed according to the case number which has been calculated from the input data (j,k).

MATRX

The determinant of a (4×4) matrix is solved for the linearized constraint equations.

ELIM

A variable is eliminated between two general conics to obtain a quartic polynomial using Bezout's method [23]. The resulting quartic polynomial gives the x coordinates of the circle point solutions for the five-position synthesis in the moving reference system. Coefficients are rearranged for the straight line problem and the same equations give

a cubic polynomial for the x coordinate of the center point solutions in the fixed reference system.

QUARTC

A quartic polynomial is rooted in closed form (real roots only).

CUBIC

A cubic polynomial is rooted in closed form. The resolvent cubic is solved for the Subroutine QUARTC.

COG

Path cognates are computed for a four-link mechanism and the solutions are written. Analysis and Grashof criteria subroutines are called.

ANALYS

All the dyads of a ten-link path cognates (Robert's cognates) mechanism are analyzed for the number of positions and time derivatives specified depending on the case number. The specified path properties for synthesis are applied at the coupler point and the angular properties are found. This subroutine provides analysis support required for synthesis of multi-link mechanisms and verifies the synthesis results. This routine is called only by COG (COGS must be chosen as an option).

INV2

Inversion of a 2×2 matrix is performed for Subroutine ANALYS.

GRASHF

A four-link chain is evaluated according to Grashof criteria. Link length and modified link length ratios are calculated.

FOUR

All cases of four-position synthesis are solved by utilizing Subroutine EQUATN. Four-position solutions are written.

DET

The determinant of a (3×3) matrix is solved to service Subroutine FOUR.

CRVTRN

This routine calculates the fixed pivot or center point solutions for the four-position problem and transforms the circle points to the fixed reference system.

INVERT

Input specifications for five- and six-link synthesis and four-link function generation are converted to specifications for constraint link synthesis by inversion and analysis. The four- or five-position synthesis routines are called by using these specifications.

A listing of MECSYN along with examples of input and output follows.

```
C     ************************MECSYN*****************************************
C     *FOUR AND FIVE POSITION MODULAR SYNTHESIS OF MULTI-LINK                 *
C     *PLANAR MECHANISMS FOR TIME DEPENDENT AND GEOMETRIC KINEMATIC           *
C     *SPECIFICATIONS (ALL COMBINATIONS OF POSITION AND DERIVATIVES)          *
C     *REFERENCE   FIVE POSITION SYNTHESIS OF SIX-LINK MECHANISMS             *
C     *COORDINATING UP TO FOUR MOTION PARAMETERS,  U.F. MASTER'S              *
C     *THESIS, AUGUST 1971 AND  SYNTHESIS OF MULTI-LINK MECHANISMS            *
C     *FOR DYNAMIC SPECIFICATIONS,  U.F. PH.D. DISSERTATION, AUGUST           *
C     *1974 - ARVID MYKLEBUST     UNIVERSITY OF FLORIDA                      *
C     *LAST REVISED - JULY 1974                                               *
C     ***********************************************************************
      DIMENSION U(10),V(10),I(12),KR(3),IC(10)
      DOUBLE PRECISION DL,DR,FOLD,CZ,XBPT,YBPT,XBCT,YBCT,AP,BP,P,
     1U,V,DET,XBP,YBP,DETLS
      COMMON DL(4,4),DR(3,4),FOLD(4,4),CZ(3,4),XBPT(4),YBPT(4),XBCT(4),
     *YBCT(4),AP(5,5),BP(5,5),P(5,5),STLN,KRR,NCASE/UNIT/NI,NO
      COMMON /COM1/ISTART,ISTOP,ISTEP,NPOS
      DATA I/2,1,3,1,3,2,4,1,4,2,4,3/,KR/2,6,12/
      NI=5
      NO=6
    1 CALL INPUT(TRACE,IEND,CGN,CHECK,IC)
      IF(IEND.EQ.1) GO TO 2
      IF(NPOS.EQ.0.OR.NPOS.EQ.4) GO TO 1
      IF(NPOS.EQ.45)WRITE(NO,14)NCASE
   14 FORMAT(//24X,29HFIVE POSITION SOLUTION - CASE,2X,I1)
      IF (NPOS .EQ. 45) NPOS=5
C     SUBROUTINE EQUATN CALCULATES COEFFICIENTS OF CONSTRAINT EQUATIONS
      CALL EQUATN
      IF(TRACE) 9,9,3
C     OUTPUT STATEMENTS FOR INTERMEDIATE CALCULATIONS
    3 WRITE(NO,4) (DR(1,L),DR(2,L),DR(3,L),L=1,4),DL
    4 FORMAT(1H1////34X,36HRIGHT MATRIX DEFINED BY Z0,Z1 AND Z2//13X,3F2
     12.6/13X,3F22.6/13X,3F22.6/13X,3F22.6///33X,38HLEFT MATRIX DEFINED
     2BY Z3,Z4,Z5 AND Z6/4(/1X,4F22.6))
```

```
C      SOLUTIONS FOR Z3 THRU Z6 IN TERMS OF Z0, Z1, Z2 ARE DETERMINED
C      IF STRAIGHT LINE MOTION(ALPHA/BETA=ALPHA'/BETA'=...=ALPHA''''/
C      BETA''''=CONSTANT),MATRICES ARE REARRANGED AND SOLUTIONS FOR Z1
C      THRU Z4 IN TERMS OF Z0,Z5,Z6 ARE DETERMINED BECALSE LEFT MATRIX IS
C      SINGULAR. SAME FORMULATION IS USED BUT COEFFICIENTS ARE DIFFERENT.
C      BURMESTER CENTERS AND POINTS ARE REVERSED IN THE SOLUTION(CUBIC IN
C      X-CENTER RATHER THAN QUARTIC IN X-POINT).
    9  DO 50 M=1,4
       DO 90 N=1,4
   90  HOLD(N,M)=DL(N,M)
       IF(STLN.NE.1.) GO TO 50
       DO 100 L=1,2
       DL(L,M)=-DR(L+1,M)
       DL(L+2,M)=HOLD(L,M)
       DR(L+1,M)=-HOLD(L+2,M)
       HOLD(L,M)=DL(L,M)
  100  HOLD(L+2,M)=DL(L+2,M)
   50  CONTINUE
       CALL MATRX(DET)
       DETLS = DET
       IF(DETLS.EQ.0.) GO TO 120
C      LOOPS DETERMINE THE RELATIONS FOR Z3 THRU Z6.
       DO 40  M=1,4
       DO 40  J=1,3
       DO 20  K=1,4
   20  DL(M,K) = DR(J,K)
       CALL MATRX(DET)
       DZ(J,M)=DET/DETLS
       DO 40 L=1,4
   40  DL(M,L) = HOLD(M,L)
       IF (TRACE) 31,31,7
    7  WRITE(NO,8) DZ
    8  FORMAT(//18X,57HCOEFFICIENTS DEFINING Z3 THRU Z6 IN TERMS OF Z0,Z1
      1 AND Z2//(8X,3F22.6))
```

```
 31 CALL ELIM (TRACE)
    IF(STLN.NE.1.) GO TO 36
    IF(KRR.EQ.0) GO TO 37
    DO 110 N=1,KRR
    XBCT(N)=XBPT(N)
    YBCT(N)=YBPT(N)
    XBPT(N)=CZ(1,1)-CZ(2,1)*XBCT(N)-CZ(3,1)*YBCT(N)
110 YBPT(N)=CZ(1,2)-CZ(2,2)*XBCT(N)-CZ(3,2)*YBCT(N)
    GO TO 37
 36 DO 30 N=1,KRR
    XBCT(N) = -(CZ(1,3) + CZ(2,3)*XBPT(N) + CZ(3,3)*YBPT(N))
 30 YBCT(N) = -(CZ(1,4) + CZ(2,4)*XBPT(N) + CZ(3,4)*YBPT(N))
 37 WRITE(NO,11)
 11 FORMAT(//23X,35HCOORDINATES OF THE CONSTRAINT LINKS//17X,6HPOINTS,
   135X,7HCENTERS/)
    IF(KRR)32,32,34
 32 WRITE(NO,33)
 33 FORMAT(///34X,23HNO REAL SOLUTIONS EXIST)
    GO TO 6
 34 DO 60 L=1,KRR
    XBP=XBPT(L)
    YBP=YBPT(L)
    XBPT(L)=XBP*DCOS(P(1,1))-YBP*DSIN(P(1,1))+AP(1,1)
    YBPT(L)=XBP*DSIN(P(1,1))+YBP*DCOS(P(1,1))+BP(1,1)
 60 WRITE (NO,35) XBPT(L) , YBPT(L) , XBCT(L) , YBCT(L)
 35 FORMAT(2X, 4HX = ,F12.5, 5X, 4HY = ,F12.5,  5X, 4HX = ,F12.5
   *, 5X, 4HY = ,F12.5    )
  6 IF(CGN.NE.1..OR.KRR.LT.2) GO TO 13
    U(5)=AP(1,1)
    V(5)=BP(1,1)
    WRITE(NO,5) U(5),V(5)
  5 FORMAT(1H1,49X,13HCOUPLER POINT/46X,1H(,F8.3,2H, ,F8.3,1H))
    K=KR(KRR-1)
    DO 80 J=2,K,2
```

```
      DO 70  L=1,2
      U(L)=XBCT(I(J-L+1))
      U(L+2)=XBPT(I(J-L+1))
      V(L)=YBCT(I(J-L+1))
   70 V(L+2)=YBPT(I(J-L+1))
   80 CALL COG(U,V,CHECK,IC,NCASE,AP,BP)
      GO TO 13
  120 WRITE(NO,130)
  130 FORMAT(10X,18HMATRIX IS SINGULAR/10X,67H(MOTION IS STRAIGHT LINE O
     UR HAS MORE THAN THREE PARALLEL POSITIONS))
   13 GO TO 1
    2 STOP
      END
      SUBROUTINE INPUT(TRACE,IEND,CGN,CHECK,IC)
      DIMENSION IC(10),JCASE(11),KCASE(7),KEY(10),IOPT(6),H(2),IK(6),
     1NCASE(7),U(10),V(10)
      DOUBLE PRECISION DL,OR,HOLD,CZ,XBPT,YBPT,XBCT,YBCT,AP,BP,P,
     1A,B,G,U,V
      COMMON DL(4,4),OR(3,4),HOLD(4,4),CZ(3,4),XBPT(4),YBPT(4),XBCT(4),
     *YBCT(4),AP(5,5),BP(5,5),P(5,5),STLN,KRR,NCASE/UNIT/NI,NO
      COMMON /COM1/ISTART,ISTOP,ISTEP,NPOS
      DATA JCASE/1,2,3,4,5,0,6,0,0,0,7/,KCASE/1,2,3,4,0,0,5/,
     2KEY/4HCOGS,4HTRCE,4HANAL,4HSTLN,4HFGEN,4HSIXL,4HFOUR,4HFREV,
     34HCCGC,4H    /,H/4H  NO,4H YES/,IB/4H    /,MCASE/1,1,2,2,3,4,5/
C     SUBROUTINE READS AND WRITES THE INPUT DATA
C     FIVE POSITION CASES
CASE1,PPPPP  2,PPPP-P  3,PPP-PP  4,PPP-P-P  5,PP-PP-P  6,PP-P-P-P  7,P-P-P-P-P
C     FOUR POSITION CASES
CASE1,PPPP   2,PPP-P   3,PP-PP   4,PP-P-P   5,P-P-P-P
C     ALL OPTIONS BLANK = NO
C     COGS    COGNATE MECHANISMS, GRASHOF CRITERION, LINK LENGTH RATIOS
C     TRCE    INTERMEDIATE PRINTING OF EQUATIONS
C     ANAL    ANALYSIS OF ANGULAR PROPERTIES-REQUIRES COGNATES ALSO
C     STLN    STRAIGHT LINE SPECIFICATIONS ROUTINE FOR FIVE TMSP
```

```
C      FGEN    INPUT PARAMETER CONVERSION, FOUR-BAR FUNCTION GENERATION
C      SIXL    INPUT PARAMETER CONVERSION FOR SYNTHESIS OF FIVE AND SIX-LINK
C              MECHANISMS BY INVERSION(INCLUDING SIX-LINK FUNCTION GEN.)
C      FOUR    FOUR POSITION PROBLEM ONLY-FIVE POSITION BY DEFAULT
C      FRFV    FOUR AND FIVE POSITION SOLUTIONS
C              FOUR POSITION PROBLEM USES FIRST FOUR OF FIVE SPECS
C              FOUR POSITION CASES                      1    2   3  4  5
C              CORRESPONDING FIVE POSITION CASES    1,2  3,4  5  6  7
C      START   INITIAL VALUE OF ANGLE-FOUR POSITION SWEEP-DEFAULT=1
C      STOP    FINAL VALUE OF ANGLE-FOUR POSITION SWEEP-DEFAULT=176
C      STEP    INCREMENT OF ANGLE-FOUR POSITION SWEEP-DEFAULT=5
C      XBPT    TEMPORARY STORAGE OF 32 CHARACTER TITLE
       IEND=0
       NPOS=5
       ICASE=0
       I=5
       DGN=0.
       TRACE=0.
       CHECK=0.
       STLN=0.
       FNGN=0.
       SIXINV=0.
       CCGC=0.
       DO 50 L=1,6
       ICPT(L)=I8
   50  IK(L)=1
       READ(NI,6,END=8,ERR=9)(IOPT(L),L=1,6),XBPT,START,STOP,STEP
    6  FORMAT(6(A4,1X),4A8,3X,3F5.0)
       DO 20 L=1,6
       DO 30 KL=1,10
       IF(ICPT(L).NE.KEY(KL)) GO TO 30
       GO TO (21,22,23,24,25,26,27,28,29,20),KL
   30  CONTINUE
       WRITE(NO,53)IOPT(L)
```

```
53 FORMAT(5X,18H***** OPTION WORD ,A4,21H NOT RECOGNIZED *****)
   GO TO 20
21 CGN=1.
   IK(1)=2
   GO TO 20
22 TRACE=1.
   IK(2)=2
   GO TO 20
23 CHECK=1.
   IK(3)=2
   GO TO 20
24 STLN=1.
   IK(4)=2
   GO TO 20
25 FNGN=1.
   IK(5)=2
   GO TO 20
26 SIXINV=1.
   IK(6)=2
   GO TO 20
27 NPCS=4
   I=4
   GO TO 20
28 NPCS=45
   I=5
   GO TO 20
29 CCGC=1.
   IK(1)=2
20 CONTINUE
   ISTART=IFIX(START)
   ISTOP=IFIX(STOP)
   ISTEP=IFIX(STEP)
   IF(ISTART.EQ.0) ISTART=1
   IF(ISTOP.EQ.0) ISTOP=171
```

```
      IF(ISTEP.EQ.0) ISTEP=10
      IF(CCGD.NE.1.)GC TO 51
      READ(NI,52)U(1),V(1),U(3),V(3),U(2),V(2),U(4),V(4),U(5),V(5)
   52 FORMAT(4F10.0/4F10.0/2F10.0)
      NPCS=0
      GC TO 15
   51 IF(FNGN.EQ.1..OR.SIXINV.EQ.1.).AND.NPOS.NE.4)GO TO 15
      CO 2 L=1,I
      READ(NI,7)J,K,A,B,G
    7 FORMAT(2I1,8X,3F10.4)
      ICASE=ICASE+J
      IC(L)=J
      IC(L+5)=K
      N=J+1
      M=K+1
      AP(N,M)=A
      BP(N,M)=B
    2 P(N,M)=G
      NCASE=JCASE(ICASE+1)
      IF(NPOS.EQ.4)NCASE=KCASE(ICASE+1)
      IF(NPCS.EQ.45)NCASE=MCASE(NCASE)
   15 WRITE(NO,1)(H(IK(L)),L=1,6),NPOS,XBPT
    1 FORMAT(1H1//  9X,51H**************************************************
     1*****/9X,1H*,21X,6HMECSYN,22X,1H*/
     1                    9X,51H*MODULAR SYNTHESIS OF MULTI-LINK PLANAR
     2MECHANISMS*/9X,51H* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATION
     3S */9X,51H*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    */
     49X,51H**************************************************///22X,
     57HOPTIONS/10X,27HCOGNATES, GRASHOF CRITERION/10X,
     622HAND LINK LENGTH RATIOS,9X,  A4/10X,21HINTERMEDIATE PRINTING,
     710X,  A4/10X,8HANALYSIS,23X,  A4/10X,28HSTRAIGHT LINE SPECIFICATIO
     8NS,3X,  A4/10X,30HFUNCTION GENERATION(FOUR-LINK),1X,  A4/10X,
     928HFIVE- AND SIX-LINK SYNTHESIS,3X,  A4/10X,26HFOUR OR FIVE MSP (4
     15=BOTH),7X,I2//10X,4A8//)
```

```
      IF(CCGO.NE.1.)GO TO 54
      WRITE(NO,55)U(5),V(5)
   55 FORMAT(50X,13HCOUPLER POINT/46X,1H(,F8.3,2H, ,F8.3,1H))
      CALL COG(U,V,CHECK,IC,NCASE,AP,BP)
      GO TO 4
   54 IF(FNGN.EQ.1..OR.SIXINV.EQ.1.).AND.NPOS.NE.4)
     1CALL INVERT(NCASE,FNGN,SIXINV,IEND,NPOS,AP,BP,P,IC,ICASE)
      IF((FNGN.EQ.1..OR.SIXINV.EQ.1.).AND.NPOS.EQ.45)NCASE=MCASE(NCASE)
      IF(FNGN.EQ.1..OR.SIXINV.EQ.1.).AND.NPOS.NE.4) GO TO 12
      WRITE(NO,13) NCASE
   13 FORMAT(20X,24HSPECIFICATIONS - CASE    ,I1//2X,9HPOS. DER.,4X,
     111HX COMPONENT,5X,11HY COMPONENT,8X,5HTHETA/)
   12 DO 3 L=1,I
      N=IC(L)+1
      M=IC(L+5)+1
      WRITE(NO,14)IC(L),IC(L+5),AP(N,M),BP(N,M),P(N,M)
   14 FORMAT(3X,I1,4X,I1,3(4X,F12.5))
    3 IF(M.EQ.1) P(N,M)=P(N,M)/57.295779513100
      IF(NPOS.NE.5) GO TO 11
    4 RETURN
    8 IEND=1
      RETURN
   11 CALL FOUR
      IF(NPOS.EQ.45)NCASE=JCASE(ICASE+1)
      RETURN
    9 WRITE(NO,10)
   10 FORMAT(1H ,5X,43HCHECK THE  OPTIONS  RECORD FOR INPUT ERRORS//)
      RETURN
      END
      SUBROUTINE EQUATN
      DIMENSION A(7),B(5)
      DOUBLE PRECISION DL,DR,HOLD,CZ,XBPT,YBPT,XBCT,YBCT,AP,BP,P,
     1C,S
      COMMON DL(4,4),DR(3,4),HOLD(4,4),CZ(3,4),XBPT(4),YBPT(4),XBCT(4),
```

```
     *YBCT(4),AP(5,5),BP(5,5),P(5,5),STLN,KRR,NCASE
      COMMON /COM1/ISTART,ISTOP,ISTEP,NPOS
      INTEGER A,B
      DATA A/1,2,2,3,3,4,5/,B/2,4,5,6,7/
C     SUBRCUTINE CALCULATES COEFFICIENTS OF THE CONSTRAINT EQUATIONS
C     FOR DERIVATIVES WITH RESPECT TO A OUMMY PARAMETER, TAU.
C     TIME DEPENDENT SPECS, TAU=TIME, SPECIFY ALL TIME DERIVATIVES
C     GEOMETRIC SYNTHESIS, TAU=PHI, O PHI/O TAU =1,ALL OTHER OER.=0
C     SPECIFICATIONS FOR SECOND OERIVATIVE AND HIGHER MUST BE MADE FOR
C     THE FIRST POSITION ONLY(J=0), WITHOUT LOSS OF GENERALITY.
      N=0
      M=0
      S=DSIN(P(1,1))
      C=DCOS(P(1,1))
      IF (NPOS .NE. 5) NCASE=B(NCASE)
      NFINPT=A(NCASE)
      IF (NPOS .NE. 5) NFINPT=NFINPT-1
      IF(NFINPT.EQ.1) GO TO 12
      DO 10 M=2,NFINPT
      HOLD(M,1)=DSIN(P(M,1))
      HOLD(M,2)=DCOS(P(M,1))
      N = M - 1
      DR(1,N) = -(AP(M,1)**2 + BP(M,1)**2)/2.0
     1+(AP(1,1)**2+BP(1,1)**2)/2.
      DR(2,N)=-(AP(M,1)*HOLD(M,2)  +BP(M,1)*HOLD(M,1)  )
     **+AP(1,1)*C           +BP(1,1)*S
      DR(3,N)=-(BP(M,1)*HOLD(M,2)  -AP(M,1)*HOLD(M,1)
     *-(BP(1,1)*C           -AP(1,1)*S))
      DL(1,N)=HOLD(M,2)-C
      DL(2,N)=HOLD(M,1)-S
      DL(3,N)=AP(M,1)-AP(1,1)
   10 DL(4,N)=BP(M,1)-BP(1,1)
      IF((NPOS.NE.5.AND.N.EQ.3).OR.N.EQ.4)GO TO 100
   12 NDER = 1
```

```
      IF(NCASE - 3) 13,14,13
   13 IF(NCASE -5) 15,14,15
   14 NDER=2
   15 M=0
      DO 16 J=1,NDER
      N = N + 1
      M = M + 1
      CR(1,N) = -AP(M,1)*AP(M,2) - BP(M,1)*BP(M,2)
      CR(2,N)=-((-AP(M,1)*P(M,2)+BP(M,2))*DSIN(P(M,1))
     *+(AP(M,2)+BP(M,1)*P(M,2))*DCOS(P(M,1)))
      CR(3,N)=-((-AP(M,2)-BP(M,1)*P(M,2))*DSIN(P(M,1))
     *+(-AP(M,1)*P(M,2)+BP(M,2))*DCOS(P(M,1)))
      CL(1,N)=-P(M,2)*DSIN(P(M,1))
      CL(2,N)=P(M,2)*DCOS(P(M,1))
      CL(3,N) = AP(M,2)
   16 CL(4,N) = BP(M,2)
      IF((NPOS.NE.5.AND.N.EQ.3).OR.N.EQ.4) GO TO 100
   17 N=N+1
      CR(1,N)=-AP(1,2)**2-BP(1,2)**2-AP(1,1)*AP(1,3)-BP(1,1)*BP(1,3)
      CR(2,N)=-((-AP(1,1)*P(1,3)-2*AP(1,2)*P(1,2)-BP(1,1)*P(1,2)*P(1,2)
     *+BP(1,3))*S           +(-AP(1,1)*P(1,2)**2+AP(1,3)+BP(1,1)*P(1,3)
     *+2*BP(1,2)*P(1,2))*C)
      CR(3,N)=-((AP(1,1)*P(1,2)**2-AP(1,3)-BP(1,1)*P(1,3)
     *-2*BP(1,2)*P(1,2))*S           +(-AP(1,1)*P(1,3)-2*AP(1,2)*
     *P(1,2)-BP(1,1)*P(1,2)**2+BP(1,3))*C)
      CL(1,N)=-P(1,3)*S           -P(1,2)*P(1,2)*C
      CL(2,N)=-P(1,2)*P(1,2)*S           +P(1,3)*C
      CL(3,N) = AP(1,3)
      CL(4,N) = BP(1,3)
      IF((NPOS.NE.5.AND.N.EQ.3).OR.N.EQ.4) GO TO 100
   18 N=N+1
      CR(1,N)=-(AP(1,1)*AP(1,4)+3*AP(1,2)*AP(1,3)+BP(1,1)*BP(1,4)+3*BP(1
     *,2)*BP(1,3))
      CR(2,N)=-((AP(1,1)*P(1,2)**3-AP(1,1)*P(1,4)-3*AP(1,2)*P(1,3)
```

```
     *-3*AP(1,3)*P(1,2)-3*BP(1,1)*P(1,2)*P(1,3)-3*BP(1,2)*P(1,2)**2
     **+BP(1,4))*S            +(-3*AP(1,1)*P(1,2)*P(1,3)-3*AP(1,2)*
     *P(1,2)**2+AP(1,4)-BP(1,1)*P(1,2)**3+BP(1,1)*P(1,4)+3*BP(1,2)*
     *P(1,3)+3*BP(1,3)*P(1,2))*C)
      CR(3,N)=-((AP(1,1)*P(1,2)**3-AP(1,1)*P(1,4)-3*AP(1,2)*P(1,3)
     *-3*AP(1,3)*P(1,2)-3*BP(1,1)*P(1,2)*P(1,3)-3*BP(1,2)*P(1,2)**2
     **+BP(1,4))*C            -(-3*AP(1,1)*P(1,2)*P(1,3)-3*AP(1,2)*
     *P(1,2)**2+AP(1,4)-BP(1,1)*P(1,2)**3+BP(1,1)*P(1,4)+3*BP(1,2)*
     *P(1,3)+3*BP(1,3)*P(1,2))*S)
      CL(1,N)=(P(1,2)**3-P(1,4))*S           -3*P(1,2)*P(1,3)*C
      CL(2,N)=(-3*P(1,2)*P(1,3))*S           +(-P(1,2)**3+P(1,4))*C
      CL(3,N) = AP(1,4)
      CL(4,N) = BP(1,4)
      IF (NPOS.NE.5.OR.N.EQ.4)GO TO 100
   19 N=N+1
      CR(1,N)=-(AP(1,1)*AP(1,5)+4*AP(1,2)*AP(1,4)+3*AP(1,3)**2+BP(1,1)*B
     *P(1,5)+4*BP(1,2)*BP(1,4)+3*BP(1,3)**2)
      CR(2,N)=-((-AP(1,1)*P(1,5)+6*AP(1,1)*P(1,3)*P(1,2)**2
     **+4*AP(1,2)*P(1,2)**3-4*AP(1,2)*P(1,4)-6*AP(1,3)*P(1,3)
     *-4*AP(1,4)*P(1,2)+BP(1,1)*P(1,2)**4-4*BP(1,1)*P(1,2)*P(1,4)
     *-3*BP(1,1)*P(1,3)**2-12*BP(1,2)*P(1,2)*P(1,3)-6*BP(1,3)
     **P(1,2)**2+BP(1,5))*S            +(AP(1,1)*P(1,2)**4-4*AP(1,1)*
     *P(1,2)*P(1,4)-3*AP(1,1)*P(1,3)**2-12*AP(1,2)*P(1,2)*P(1,3)
     *-6*AP(1,3)*P(1,2)**2+AP(1,5)+BP(1,1)*P(1,5)-6*BP(1,1)*P(1,3)*
     *P(1,2)**2-4*BP(1,2)*P(1,2)**3+4*BP(1,2)*P(1,4)+6*BP(1,3)*
     *P(1,3)+4*BP(1,4)*P(1,2))*C)
      CR(3,N)=-((-AP(1,1)*P(1,2)**4+4*AP(1,1)*P(1,2)*P(1,4)+3*
     *AP(1,1)*P(1,3)**2+12*AP(1,2)*P(1,2)*P(1,3)+6*AP(1,3)*P(1,2)**2
     *-AP(1,5)-BP(1,1)*P(1,5)+6*BP(1,1)*P(1,3)*P(1,2)**2+4*BP(1,2)*
     *P(1,2)**3-4*BP(1,2)*P(1,4)-6*BP(1,3)*P(1,3)-4*BP(1,4)*P(1,2))*
     *S            +(-AP(1,1)*P(1,5)+6*AP(1,1)*P(1,3)*P(1,2)**2+4*AP(1,2)
     **P(1,2)**3-4*AP(1,2)*P(1,4)-6*AP(1,3)*P(1,3)-4*AP(1,4)*P(1,2)
     **+BP(1,1)*P(1,2)**4-4*BP(1,1)*P(1,2)*P(1,4)-3*BP(1,1)*P(1,3)**2-12
     **BP(1,2)*P(1,2)*P(1,3)-6*BP(1,3)*P(1,2)**2+BP(1,5))*C)
```

```
      CL(1,N)=(6*P(1,3)*P(1,2)**2-P(1,5))*S           +(-4*P(1,2)*
     *P(1,4)-3*P(1,3)**2+P(1,2)**4)*C
      CL(2,N)=(-3*P(1,3)**2-4*P(1,2)*P(1,4)+P(1,2)**4)*S
     **+(-6*P(1,3)*P(1,2)**2+P(1,5))*C
      CL(3,N) = AP(1,5)
      CL(4,N) = BP(1,5)
  100 RETURN
      END
      SUBROUTINE MATRX(DET)
      DOUBLE PRECISION DL,DR,HDLD,CZ,XBPT,YBPT,XBCT,YBCT,AP,BP,P,
     1DET,DNS1,DNS2,DNS3,DNS4,DPS1,DPS2,DPS3,DPS4
      COMMON DL(4,4),DR(3,4),HDLD(4,4),CZ(3,4),XBPT(4),YBPT(4),XBCT(4),
     *YBCT(4),AP(5,5),BP(5,5),P(5,5),STLN,KRR,NCASE
C     SUBROUTINE EVALUATES THE DETERMINANT OF A (4X4) MATRIX.
      DNS1 = DL(2,2)*DL(3,3)*DL(4,4) + DL(2,3)*DL(3,4)*DL(4,2) +
     1DL(2,4)*DL(4,3)*DL(3,2)
      DPS1 = DL(2,4)*DL(3,3)*DL(4,2) + DL(2,3)*DL(3,2)*DL(4,4) +
     1DL(2,2)*DL(3,4)*DL(4,3)
      DNS2 = DL(2,1)*DL(3,3)*DL(4,4) + DL(3,1)*DL(4,3)*DL(2,4) +
     1DL(2,3)*DL(3,4)*DL(4,1)
      DPS2 = DL(2,4)*DL(3,3)*DL(4,1) + DL(3,1)*DL(2,3)*DL(4,4) +
     1DL(3,4)*DL(4,3)*DL(2,1)
      DNS3 = DL(2,1)*DL(3,2)*DL(4,4) + DL(3,1)*DL(4,2)*DL(2,4) +
     1DL(2,2)*DL(3,4)*DL(4,1)
      DPS3 = DL(2,4)*DL(3,2)*DL(4,1) + DL(3,1)*DL(2,2)*DL(4,4) +
     1DL(3,4)*DL(4,2)*DL(2,1)
      DNS4 = DL(2,1)*DL(3,2)*DL(4,3) + DL(2,3)*DL(3,1)*DL(4,2) +
     1DL(2,2)*DL(3,3)*DL(4,1)
      DPS4 = DL(2,3)*DL(3,2)*DL(4,1) + DL(2,2)*DL(3,1)*DL(4,3) +
     1DL(2,1)*DL(4,2)*DL(3,3)
      DET=DL(1,1)*(DNS1-DPS1)-DL(1,2)*(DNS2-DPS2)+DL(1,3)*(DNS3-DPS3)-
     *DL(1,4)*(DNS4-DPS4)
      RETURN
      END
```

```
      SUBROUTINE ELIM(TRACE)
      DIMENSION SV(3),A(6),B(6),C(11),U(11),V(11)
      DOUBLE PRECISION DL,DR,HOLD,CZ,XBPT,YBPT,XBCT,YBCT,AP,BP,P,
     1A,B,C,C0,C1,C2,C3,C4,DL,DR,SV,DIS
      COMMON DL(4,4),DR(3,4),HOLD(4,4),CZ(3,4),XBPT(4),YBPT(4),XBCT(4),
     *YBCT(4),AP(5,5),BP(5,5),P(5,5),STLN,KRR,NCASE/UNIT/NI,NO
      INTEGER U,V
      DATA U/5*1,3*2,3*3/,V/2,3,4,5,6,4,5,6,4,5,6/
C     Y IS ELIMINATED BETWEEN TWO CONICS TO OBTAIN A QUARTIC POLYNOMIAL
C     COEFFICIENTS FOR THE CONICS
      IF(STLN.NE.1.) GO TO 1
      A(1)=CZ(3,2)
      A(2)=CZ(3,1)+CZ(2,2)
      A(3)=CZ(3,3)-CZ(1,2)
      A(4)=CZ(2,1)
      A(5)=CZ(2,3)-CZ(1,1)
      A(6)=-CZ(1,3)
      B(1)=CZ(3,1)
      B(2)=CZ(2,1)-CZ(3,2)
      B(3)=CZ(3,4)-CZ(1,1)
      B(4)=-CZ(2,2)
      B(5)=CZ(1,2)+CZ(2,4)
      B(6)=-CZ(1,4)
      GO TO 2
    1 A(1)= CZ(3,4)
      A(2)= CZ(3,3) + CZ(2,4)
      A(3)= CZ(1,4) - CZ(3,1)
      A(4)= CZ(2,3)
      A(5)= CZ(1,3) - CZ(2,1)
      A(6)= -CZ(1,1)
      B(1)= CZ(3,3)
      B(2)= CZ(2,3) - CZ(3,4)
      B(3)= CZ(1,3) + CZ(3,2)
      B(4)= -CZ(2,4)
```

```
      B(5)= -CZ(1,4) + CZ(2,2)
      B(6)= CZ(1,2)
C     COEFFICIENTS DEFINED IN THE ELIMINATION
    2 DO 10 I=1,11
   10 C(I)=A(U(I))*B(V(I))-A(V(I))*B(U(I))
C     COEFFICIENTS OF THE QUARTIC IN X
      C4=C(3)*C(3)-C(1)*C(6)
      C3=2.*C(3)*C(4)-C(1)*(C(7)+C(9))-C(2)*C(6)
      C2=C(4)*C(4)+2.*C(3)*C(5)-C(1)*(C(8)+C(10))-C(2)*(C(7)+C(9))
      C1=2.*C(4)*C(5)-C(2)*(C(8)+C(10))-C(1)*C(11)
      C0=C(5)*C(5)-C(2)*C(11)
      IF(STLN.EQ.1.) C4=0.
      IF(C4) 30,90,30
   90 IF(C3) 110,120,110
  120 IF(C2) 130,140,130
  140 KRR=1
      XBPT(1)=-C0/C1
      GO TO 40
  130 DIS=(C1/(2.*C2))**2-C0/C2
      IF(DIS) 150,160,160
  150 KRR=0
      GO TO 170
  160 KRR=2
      XBPT(1)=-C1/(2.*C2)+DSQRT(DIS)
      XBPT(2)=-C1/(2.*C2)-DSQRT(DIS)
      GO TO 40
  110 CALL CUBIC(C2/C3,C1/C3,C0/C3,SV)
      DO 20 I=1,KRR
   20 XBPT(I)=SV(I)
      GO TO 40
   30 CALL QUARTC(C3/C4,C2/C4,C1/C4,C0/C4)
   40 DO 50 I=1,KRR
   50 YBPT(I)=-(C(3)*XBPT(I)**2+C(4)*XBPT(I)+C(5))/(C(1)*XBPT(I)+C(2))
  170 IF(TRACE) 100,100,60
```

```
   60 WRITE(NO,70) C4,C3,C2,C1,C0
   70 FORMAT(//30X,42HTHE QUARTIC DEFINING X IN THE MOVING PLANE//19X,1H
     14,17X,1H3,17X,1H2/4X,F12.5,4(6H(X) + ,F12.5),8H    =    0/)
  100 RETURN
      ENC
      SUBROUTINE QUARTC(B,C,D,E)
      DOUBLE PRECISION DL,CR,HOLD,CZ,XBPT,YBPT,XBCT,YBCT,AP,BP,P,
     1B,C,C,E,S,T,SV,TS,DIS1,DIS2
      DIMENSION SV(3)
      COMMON DL(4,4),CR(3,4),HOLD(4,4),CZ(3,4),XBPT(4),YBPT(4),XBCT(4),
     *YBCT(4),AP(5,5),BP(5,5),P(5,5),STLN,KRR,NCASE/UNIT/NI,NO
C     SUBROUTINE CALCULATES THE REAL ROOTS OF A QUARTIC POLYNOMIAL
C     REFERENCE-ELEMENTARY THEORY OF EQUATIONS-L.E.DICKSON,WILEY,1914
      CALL CUBIC(-C,B*D-4.*E,-B*B*E+4.*C*E-D*D,SV)
      DO 10 I=1,KRR
      S=SV(I)
      TS=B*B-4.*C+4.*S
      IF(TS)10,10,11
   10 CONTINUE
   11 IF(TS)12,13,14
   12 KRR=0
      GO TO 100
   13 KRR=4
      DO 15 I=1,4
   15 XBPT(I)=-B/4.
      GO TO 100
   14 T=DSQRT(TS)
      DIS1=((B-T)**2)/16.-S/2.+(B*S/2.-D)/T
      IF (DIS1)16,17,17
   17 XBPT(1)=-(B-T)/4.+DSQRT(DIS1)
      XBPT(2)=-(B-T)/4.-DSQRT(DIS1)
      KRR=2
      I=2
      GO TO 18
```

```
   16 KRR=0
      I=0
   18 DIS2=((B+T)**2)/16.-S/2.-(B*S/2.-D)/T
      IF (DIS2)100,20,20
   20 XBPT(I+1)=-(B+T)/4.+DSQRT(DIS2)
      XBPT(I+2)=-(B+T)/4.-DSQRT(DIS2)
      KRR=KRR+2
  100 RETURN
      END
      SUBROUTINE CUBIC (A1,A2,A3,SV)
      DIMENSION SV(3)
      DOUBLE PRECISION DL,DR,HOLD,CZ,XBPT,YBPT,XBCT,YBCT,AP,BP,P,
     1D,Q,R,S,T,A1,A2,A3,SD,SQ,SV,PHI
      COMMON DL(4,4),DR(3,4),HOLD(4,4),CZ(3,4),XBPT(4),YBPT(4),XBCT(4),
     *YBCT(4),AP(5,5),BP(5,5),P(5,5),STLN,KRR,NCASE
C     SUBROUTINE CALCULATES THE REAL ROOTS OF A CUBIC POLYNOMIAL
      Q=(3.*A2-A1*A1)/9.
      R=(-9.*A1*A2+27.*A3+2.*A1**3)/54.
      D=Q**3+R*R
      IF(D)10,10,11
   11 SD=DSQRT(D)
      IF(-R+SD)12,13,13
   12 S=-(DABS(-R+SD))**(1.D0/3.D0)
      GO TO 14
   13 S=(-R+SD)**(1.D0/3.D0)
   14 IF(-R-SD)15,16,16
   15 T=-(DABS(-R-SD))**(1.D0/3.D0)
      GO TO 17
   16 T=(-R-SD)**(1.D0/3.D0)
   17 SV(1)=S+T-A1/3.
      KRR=1
      GO TO 100
C     D 0, Q**3+R**2 0, -Q**3 R**2 0, -Q 0
   10 PHI=DARCOS(R/DSQRT(-Q**3))
```

```
      SQ=-2.*DSQRT(-Q)
      SV(1)=SQ*DCOS(PHI/3.)-A1/3.
      SV(2)=SQ*DCOS(PHI/3.+120./57.2957795131D0)-A1/3.
      SV(3)=SQ*DCOS(PHI/3.+240./57.2957795131D0)-A1/3.
      KRR=3
  100 RETURN
      END
      SUBROUTINE COG(U,V,CHECK,IC,NCASE,AP,BP)
      DIMENSION U(10), V(10), X(10), Y(10), R(10), P(10),IC(10),
     1AP(5,5),BP(5,5),A(8),Z(12)
      DOUBLE PRECISION P,R,S,U,V,X,Y,AP,BP
      COMMON/UNIT/NI,NO
      INTEGER A,Z
      DATA A/4,2,5,5,3,4,3,4/
      DATA Z/1,3,4,2,1,6,7,10,2,8,9,10/
      DATA X(1),Y(1)/2*0./
C     SUBROUTINE CALCULATES ROBERT'S COGNATES OF A FOUR LINK MECHANISM
C     AND CALLS ANALYSIS + GRASHOF SUBPROGRAMS
      S=DATAN2(V(2)-V(1),U(2)-U(1))
      DO 5 N=2,5
      X(N)=(U(N)-U(1))*DCOS(S)+(V(N)-V(1))*DSIN(S)
    5 Y(N)=-(U(N)-U(1))*DSIN(S)+(V(N)-V(1))*DCOS(S)
      P(1)=DATAN2(Y(3),X(3))
      R(1)=DSQRT(X(3)**2+Y(3)**2)
      DO 1 I=1,4
      P(I+1)=DATAN2(Y(A(I))-Y(A(I+4)),X(A(I))-X(A(I+4)))
    1 R(I+1)=DSQRT((Y(A(I))-Y(A(I+4)))**2+(X(A(I))-X(A(I+4)))**2)
      P(6)=P(4)-P(2)
      X(6)=R(4)*DCOS(P(4))
      Y(6)=R(4)*DSIN(P(4))
      X(7)=X(6)+(R(1)/R(2))*R(4)*DCOS(P(1)+P(6))
      Y(7)=Y(6)+(R(1)/R(2))*R(4)*DSIN(P(1)+P(6))
      X(8)=R(5)*DCOS(P(5))+X(2)
      Y(8)=R(5)*DSIN(P(5))+Y(2)
```

```
      IF(P(3)) 6,7,7
    6 P(3)=3.1415926536D0*2.-DABS(P(3))
    7 X(9)=X(5)+(R(3)/R(2))*R(4)*DCOS(P(3)+P(6))
      Y(9)=Y(5)+(R(3)/R(2))*R(4)*DSIN(P(3)+P(6))
      X(10)=(X(2)/R(2))*R(4)*DCOS(P(6))
      Y(10)=(X(2)/R(2))*R(4)*DSIN(P(6))
      DO 8 N=6,10
      U(N)=X(N)*DCOS(S)-Y(N)*DSIN(S)+U(1)
    8 V(N)=X(N)*DSIN(S)+Y(N)*DCOS(S)+V(1)
      WRITE(NO,11)U(3),V(3),U(6),V(6),U(8),V(8),U(1),V(1),U(1
     *),V(1),U(2),V(2),U(4),V(4),U(7),V(7),U(9),V(9),U(2),V(
     *2),U(10),V(10),U(10),V(10)
   11 FORMAT (1H  ,20X,18HORIGINAL MECHANISM,8X,13HFIRST COGNATE,13X,14H
     1SECOND COGNATE/20X,2HA(,F8.3, 1H,,F8.3,1H),6X,2HD(,F8.3, 1H,,F8.3,
     21H),6X,2HF(,F8.3, 1H,,F8.3,1H)/19X,3HOA(,F8.3, 1H,,F8.3,1H),5X,3HO
     3C(,F8.3, 1H,,F8.3,1H),5X,3HOF(,F8.3, 1H,,F8.3,1H)/20X,2HB(,F8.3, 1
     4H,,F8.3,1H),6X,2HE(,F8.3, 1H,,F8.3,1H),6X,2HG(,F8.3, 1H,,F8.3,1H)/1
     59X,3HOB(,F8.3, 1H,,F8.3,1H),5X,3HOE(,F8.3, 1H,,F8.3,1H),5X,3HOG(,F
     68.3, 1H,,F8.3,1H)//)
      X(5)=U(5)
      X(10)=V(5)
      DO 3 I=1,3
      DO 2 J=1,4
      X(J)=U(Z(J+4*(I-1)))
    2 X(J+5)=V(Z(J+4*(I-1)))
    3 CALL GRASHF(X,Y(I),Y(I+3),R(I),R(I+3),P(2*I-1),P(2*I))
      WRITE(NO,4) (P(I),I=1,6),(Y(I),I=1,6)
    4 FORMAT(21X,2A8,2(11X,2A8)/18X,3(18HLINK LENGTH RATIO ,F6.2,3X)/18X
     1,3(18HMODIFIED L.L.R.   ,F6.2,3X))
      IF(CHECK.EQ.0.) GO TO 10
      CALL ANALYS(U,V,AP,BP,NCASE,IC)
      WRITE(NO,12)(R(I),I=1,3)
   12 FORMAT(19X,11HLINK  A-B  ,F12.5,6X,11HLINK  D-E  ,F12.5,5X,11HLINK
     1  F-G  ,F12.5)
```

```
      WRITE(NO,13)(R(I),I=4,6)
   13 FORMAT(19X,11HLINK OA-OB ,F12.5,6X,11HLINK OD-OE ,F12.5,5X,11HLINK
     1 OF-CG ,F12.5)
      WRITE(NO,14)
   14 FORMAT(1H1)
   10 RETURN
      END
      SUBROUTINE ANALYS(U,V,AP,BP,NCASE,IC)
      DIMENSION U(10),V(10),AP(5,5),BP(5,5),IC(10),P1(5,5),P2(5,5),D(5),
     *ALPHA(5),O(5,5,6,3,2),LENGTH(6,2),HEAD(4),W(6),Z(6),S(7,2),P(12)
     *,Q(4)
      DOUBLE PRECISION D,O,U,V,X,Y,AP,BP,B1,B2,C1,C2,D1,D2,P1,P2,S1,S2,
     1ALPHA,LENGTH,HEAD
      INTEGER W,Z,S,P,Q
      DATA W/1,2,1,10,2,10/,Z/3,4,6,7,8,9/,S/1,2,2,3,3,4,5
     *,1,1,2,1,2,1,0/,P/1,3,5,1,3,5,2,4,6,2,4,6/,Q/1,2,1,2/
      DATA HEAD/8HCONSTRAI,8HCOUPLER ,8HNT LINK ,8HLINK    /
      COMMON/UNIT/NI,NO
C     SUBROUTINE CARRIES OUT A CLOSED FORM ANALYSIS FOR UP TO FOURTH
C     ORDER ANGULAR  STATE OF ALL DYADS GENERATED BY MAIN AND COG
C     ASSUMING THE COUPLER POINT OF THE DYAD HAS THE SPECIFIED STATE.
C     ANALYSIS IS DONE FOR DESIGN POSITIONS AND ALL SEVEN CASES.
C     AUTOMATICALLY(OPTION-CHECK=1.) TO ASSURE THAT SYNTHESIZED DYADS
C     AND COGNATES SATISFY THE DESIRED SPECIFICATIONS.
      DO 10 N=1,6
      DO 10 M=1,3,2
      C1=DSQRT((U(W(N))-U(Z(N)))**2+(V(W(N))-V(Z(N)))**2)
      C2=DSQRT((U(5)-U(Z(N)))**2+(V(5)-V(Z(N)))**2)
      K1=S(NCASE,1)
      DO 30 J=1,K1
      C(J)=DSQRT((AP(J,1)-U(W(N)))**2+(BP(J,1)-V(W(N)))**2)
      ALPHA(J)=DATAN2(BP(J,1)-V(W(N)),AP(J,1)-U(W(N)))
      B1=DARCOS((D1*D1-D2*D2+C(J)*D(J))/(2.*D1*D(J)))
      B2=DARCOS((D1*D1-C2*D2-D(J)*D(J))/(2.*D2*D(J)))
```

```
      P1(J,1)=ALPHA(J)+(2-M)*B1
   30 P2(J,1)=ALPHA(J)+(2-M)*B2
      IF (NCASE.EQ.7) GO TO 50
      K2=S(NCASE,2)
      DO 40 J=1,K2
   40 CALL INV2(AP(J,2),BP(J,2),P1(J,2),P2(J,2),-D1*DSIN(P1(J,1)),D2*
     *DSIN(P2(J,1)),D1*DCOS(P1(J,1)),-D2*DCOS(P2(J,1)))
      C1=DCOS(P1(1,1))
      C2=DCOS(P2(1,1))
      S1=DSIN(P1(1,1))
      S2=DSIN(P2(1,1))
      IF(NCASE.GT.4) GO TO 50
      X=AP(1,3)+(D1*P1(1,2)**2)*C1-(D2*P2(1,2)**2)*C2
      Y=BP(1,3)+(D1*P1(1,2)**2)*S1-(D2*P2(1,2)**2)*S2
      CALL INV2(X,Y,P1(1,3),P2(1,3),-D1*S1,D2*S2,D1*C1,-D2*C2)
      IF(NCASE.GT.2) GO TO 50
      X=AP(1,4)+2*D1*P1(1,2)*P1(1,3)*C1-2*D2*P2(1,2)*P2(1,3)*C2+D1*P1(1,
     *2)*(-S1*P1(1,2)**2+P1(1,3)*C1)-D2*P2(1,2)*(-S2*P2(1,2)**2+P2(1,3)*
     *C2)
      Y=BP(1,4)+2*D1*P1(1,2)*P1(1,3)*S1-2*D2*P2(1,2)*P2(1,3)*S2+D1*P1(1,
     *2)*(P1(1,3)*S1+C1*P1(1,2)**2)-D2*P2(1,2)*(P2(1,3)*S2+C2*P2(1,2)**2
     *)
      CALL INV2(X,Y,P1(1,4),P2(1,4),-D1*S1,D2*S2,D1*C1,-D2*C2)
      IF(NCASE.GT.1) GO TO 50
      X=AP(1,5)+3*D1*P1(1,2)*P1(1,4)*C1-3*D2*P2(1,2)*P2(1,4)*C2+3*D1*P1(
     *1,3)*(-S1*P1(1,2)**2+P1(1,3)*C1)-3*D2*P2(1,3)*(-S2*P2(1,2)**2+P2(1
     *,3)*C2)+C1*P1(1,2)*(-3*P1(1,2)*P1(1,3)*S1+(P1(1,4)-P1(1,2)**3)*C1)
     *+C2*P2(1,2)*(3*P2(1,2)*P2(1,3)*S2+(-P2(1,4)+P2(1,2)**3)*C2)
      Y=BP(1,5)+3*D1*P1(1,2)*P1(1,4)*S1-3*D2*P2(1,2)*P2(1,4)*S2+3*D1*P1(
     *1,3)*(P1(1,3)*S1+C1*P1(1,2)**2)-3*D2*P2(1,3)*(P2(1,3)*S2+C2*P2(1,2
     *)**2)+C1*P1(1,2)*((P1(1,4)-P1(1,2)**3)*S1+3*P1(1,2)*P1(1,3)*C1)+
     *D2*P2(1,2)*((-P2(1,4)+P2(1,2)**3)*S2-3*P2(1,2)*P2(1,3)*C2)
      CALL INV2(X,Y,P1(1,5),P2(1,5),-D1*S1,D2*S2,D1*C1,-D2*C2)
   50 DO 20 J=1,K1
```

```
      P1(J,1)=P1(J,1)*57.295779513100
      P2(J,1)=P2(J,1)*57.295779513100
      IF(P1(J,1).LT.0.) P1(J,1)=360.+P1(J,1)
      IF(P2(J,1).LT.0.) P2(J,1)=360.+P2(J,1)
      DO 20 K=1,5
      O(J,K,N,M,1)=P1(J,K)
   20 O(J,K,N,M,2)=P2(J,K)
      LENGTH(N,1)=D1
   10 LENGTH(N,2)=D2
    1 FORMAT(//42X,30HANALYSIS OF ANGULAR PROPERTIES/)
    2 FORMAT(22X,3(2A8,13X))
    3  FORMAT(2X,16X,3(22HCLOSURE 1    CLOSURE 2,7X)/1X,10HPOS. DER. )
    4 FORMAT(2X,I1,4X,I1,3X,3(3X,2F13.5))
    5 FORMAT(18X,3(11HLINK LENGTH,F11.5,7X)/)
      WRITE(NO,1)
      DO 60 K=1,4
      WRITE(NO,2)HEAD(Q(K)),HEAD(Q(K)+2),HEAD(Q(K)),
     1HEAD(Q(K)+2),HEAD(Q(K)),HEAD(Q(K)+2)
      WRITE(NO,3)
      DO 70 J=1,5
      I=IC(J)+1
      L=IC(J+5)+1
   70 WRITE(NO,4)IC(J),IC(J+5),O(I,L,P(3*K-2),1,Q(K)),O(I,L,P(3*K-2),3,Q
     *(K)),O(I,L,P(3*K-1),1,Q(K)),O(I,L,P(3*K-1),3,Q(K)),O(I,L,P(3*K),1,
     *Q(K)),O(I,L,P(3*K),3,Q(K))
   60 WRITE(NO,5)LENGTH(P(3*K-2),Q(K)),LENGTH(P(3*K-1),Q(K)),LENGTH(P(3*
     *K),Q(K))
      RETURN
      END
      SUBROUTINE INV2(X,Y,P1,P2,A,B,C,D)
      DOUBLE PRECISION A,B,C,D,X,Y,P1,P2
      P1=(D*X-B*Y)/(A*D-C*B)
      P2=(-C*X+A*Y)/(A*D-C*B)
      RETURN
```

```
      END
      SUBROUTINE GRASHF(X,LLR,LLRM,L2,L4,H1,H2)
      DIMENSION X(10),LINK(6),HEAD(8),I(12)
      DOUBLE PRECISION X,H1,H2,L2,L4,SM,AVG,LLR,HEAD,LINK,LLRM,LONG,
     1SHORT
      DATA HEAD/8H  CRANK-,8HROCKER  ,8H  DOUBLE,8H-CRANK  ,8H  DOUBLE,
     18H-ROCKER ,8HCOLLINEA,8HR LINKS /
      DATA I/1,2,3,4,5,5,2,3,4,1,2,3/
C     SUBROUTINE EVALUATES FOUR-BAR ACCORDING TO GRASHOF CRITERION AND
C     CALCULATES CHAIN LINK LENGTH RATIO AND MODIFIED LINK LENGTH RATIO
C     (USING AVERAGE LENGTH OF THE TWO RAYS TO THE COUPLER POINT AND THE
C     FOUR CHAIN LINK LENGTHS).
      SM=0.
      DO 10 J=1,6
   10 LINK(J)=DSQRT((X(I(J))-X(I(J+6)))**2+(X(I(J)+5)-X(I(J+6)+5))**2)
      L2=LINK(2)
      L4=LINK(4)
      SHORT=LINK(1)
      IS=1
      LONG=LINK(1)
      IL=1
      DO 20 J=1,4
      IF(LINK(J)-SHORT)1,2,2
    1 SHORT=LINK(J)
      IS=J
    2 IF(LINK(J)-LONG)20,20,3
    3 LONG=LINK(J)
      IL=J
   20 SM=SM+LINK(J)
      IF(2.*(LONG+SHORT)-SM)4,5,6
    4 IF(IS.EQ.1.OR.IS.EQ.3) GO TO 7
      IF(IS.EQ.4) GO TO 8
      IF(IS.EQ.2) GO TO 6
      GO TO 50
```

```
  7 H1=HEAD(1)
    H2=HEAD(2)
    GO TO 50
  8 H1=HEAD(3)
    H2=HEAD(4)
    GO TO 50
  6 H1=HEAD(5)
    H2=HEAD(6)
    GO TO 50
  5 H1=HEAD(7)
    H2=HEAD(8)
 50 LLR=LONG/SHORT
    AVG=(LINK(5)+LINK(6))/2.
    IF(SHORT.GT.AVG) SHORT=AVG
    IF(LONG.LT.AVG) LONG=AVG
    LLRM=LONG/SHORT
    RETURN
    END
    SUBROUTINE FOUR
    DIMENSION J(76),A(7,4),H(8)
    DOUBLE PRECISION A,U,BO,C2,DR,SV,X2,Y2,OY2,YPT,H,P,AP,CZ,C3,XC,YC
   1,CCO,OX1,SCO,YYF,AO,CO,C4,XF,YF,OX2,XPT,T,BP,C1,OL,X1,Y1,DIS,
   2CY1,XXF,HOLD
    COMMON /COM1/ISTART,ISTOP,ISTEP,NPOS/UNIT/NI,NO
    COMMON /COM2/A,XF,YF,AO,BO,CCO,SCO
    COMMON DL(4,4),CR(3,4),HOLD(4,4),CZ(3,4),XPT (4),YPT (4),XC  (4),
   *YC  (4),AP(5,5),BP(5,5),P(5,5),SV(3),KRR,NCASE
    DATA J/0,0,2,0,0,1,0,1,2,5,5,0,2,2,0,1,1,2,1,5,6,5,5,3,5,5,6,3,4,3
   *,3,5,6,3,3,5,3,3,6,4,6,3,6,6,4,4,6,2,1,4,3,4,4,6,3*4,8,3*7,3*6,4*5
   *,3*4,3*3,2,1/
    DO 9 I=1,8
  9 H(I)=0.
    AO=AP(1,1)
    BO=BP(1,1)
```

```
      CO=P(1,1)
      CCO=DCOS(CO)
      SCO=DSIN(CO)
      CALL EQUATN
      DO 70 I=1,3
      DO 80 K=1,3
   80 A(K,I+1)=-DR(K,I)
      DO 70 K=1,4
   70 A(K+3,I+1)=DL(K,I)
      DO 20 I=1,19
   20 CALL DET(J(I),J(I+19),J(I+38),A,H,J(I+57))
      XF= (H(1)*H(4)-H(1)*H(3)-H(2)*H(5))/2./(H(1)**2+H(2)**2)
      YF=-(H(1)*H(5)-H(2)*H(3)+H(2)*H(4))/2./(H(1)**2+H(2)**2)
      XXF=XF*CCO-YF*SCO+AO
      YYF=XF*SCO+YF*CCO+BO
      WRITE(NO,6) XXF,YYF
    6 FORMAT(1H1///5X,32HFOUR MULTIPLY SEPARATED POSITION/
     111X,19HKINEMATIC SYNTHESIS//5X,
     217HFOCAL COORDINATES,3H  (,E12.5,1H,,E12.5,1H))
      WRITE(NO,100)ISTART,ISTOP,ISTEP,NPOS
  100 FORMAT(1HC,6X,5HSTART,4X,4HSTOP,7X,4HSTEP,7X,4HNPOS/4I10)
      WRITE(NO,8)
    8 FORMAT(1HO//10X,13HCIRCLE POINTS,13X,
     113HCENTER POINTS/)
      C4=H(1)*(XF**3+XF*YF**2)+H(2)*(YF**3+YF*XF*XF)+H(3)*XF*XF+H(4)*YF
     .   **2+H(6)*XF+H(7)*YF+H(5)*XF*YF+H(8)
    7 DO 2 IT=ISTART,ISTOP,ISTEP
      T=DFLOAT(IT-1)*.017453293
      U=DTAN(T)
      C1=H(1)+U*H(2)+U*U*H(1)+U**3*H(2)
      C2=3.*H(1)*XF+H(2)*YF+H(3)+U*(2.*H(1)*YF+2.*H(2)*XF+H(5))
     .   +U*U*(H(1)*XF+3.*H(2)*YF+H(4))
      C3=3.*H(1)*XF*XF+H(1)*YF*YF+2.*H(2)*XF*YF+2.*H(3)*XF+H(6)+H(5)*YF
     .   +U*(2.*H(1)*XF*YF+3.*H(2)*YF*YF+H(2)*XF*XF+2.*H(4)*YF+H(7)+H(5)
```

```
     .     *XF)
           DIS=C2**2-4.*C1*C3
           IF(DIS) 2,5,5
         5 X1=(-C2+DSQRT(DIS))/2./C1
           X2=(-C2-DSQRT(DIS))/2./C1
           Y2=U*X2
           Y1=U*X1
           CALL CRVTRN (X1,Y1,OX1,OY1)
           CALL CRVTRN (X2,Y2,OX2,OY2)
C          ARGUMENTS BEING RETURNED ARE GIVEN IN FIXED REFERENCE
           WRITE(NO,1) X1,Y1,OX1,OY1,X2,Y2,OX2,OY2
         1 FORMAT(1H ,4F13.4)
         2 CONTINUE
       120 RETURN
           END
           SUBROUTINE DET (I,J,K,A,H,N)
           DOUBLE PRECISION A,H
           DIMENSION H(8),A(7,4)
         1 H(N)=H(N)+  A(I+1,2)*(A(J+1,3)*A(K+1,4)-A(J+1,4)*A(K+1,3))
          .      -  A(I+1,3)*(A(J+1,2)*A(K+1,4)-A(J+1,4)*A(K+1,2))
          .      +  A(I+1,4)*(A(J+1,2)*A(K+1,3)-A(J+1,3)*A(K+1,2))
           RETURN
           END
           SUBROUTINE CRVTRN(XX,YY,U,V)
           DOUBLE PRECISION A,XF,YF,AO,BO,CCO,SCO,XX,YY,U,V,X,Y,DEN,UNUM,VNUM
           COMMON/CCM2/A(7,4),XF,YF,AO,BO,CCO,SCO
           X=XX+XF
           Y=YY+YF
           DEN=(A(4,2)*X-A(5,2)*Y+A(6,2))*(A(5,3)*X+A(4,3)*Y+A(7,3))
          .   -(A(4,3)*X-A(5,3)*Y+A(6,3))*(A(5,2)*X+A(4,2)*Y+A(7,2))
           UNUM=(A(1,2)+A(2,2)*X+A(3,2)*Y)*(A(5,3)*X+A(4,3)*Y+A(7,3))
          .    -(A(1,3)+A(2,3)*X+A(3,3)*Y)*(A(5,2)*X+A(4,2)*Y+A(7,2))
           VNUM=(A(1,3)+A(2,3)*X+A(3,3)*Y)*(A(4,2)*X-A(5,2)*Y+A(6,2))
          .    -(A(1,2)+A(2,2)*X+A(3,2)*Y)*(A(4,3)*X-A(5,3)*Y+A(6,3))
```

```
      U=UNUM/DEN
      V=VNUM/DEN
      XX=X*CCO-Y*SCO+AO
      YY=X*SCO+Y*CCO+BO
      RETURN
      END
      SUBROUTINE INVERT(NC,FNGN,SIXINV,IENO,NPOS,A,B,P,IC,ICASE)
C     INPUT PARAMETER CONVERSION FOR  SYNTHESIS OF SIX-LINK AND
C     GEARED FIVE-LINK MECHANISMS(NOTATION BASED ON SYNTHESIS OF
C     STEPHENSON-2 SIX-LINK MECHANISMS)
C     ALSO FOR SYNTHESIS OF FOUR-BAR FUNCTION GENERATORS
C     TIME DEPENDENT AND GEOMETRIC KINEMATIC SPECIFICATIONS
      DIMENSION T2(5,5),T3(5,5),T4(5,5),T5(5,5),A(5,5),B(5,5),P2(5,5),
     1P3(5,5),P4(5,5),IC(10),JC(7),P(5,5),JCASE(11)
      DOUBLE PRECISION AX,AY,XC,YC,XP,YP,A,B,Z,T2,T3,T4,T5,L3,L4,A1,A2,A
     23,P2,P3,P4,S3,C3,S4,C4,P
      DATA JC/1,2,2,3,3,4,5/,JCASE/1,2,3,4,5,0,6,0,0,0,7/
      COMMON /UNIT/NI,NO
      ICASE=0
      READ(NI,1,END=110)AX,AY,XC,YC,XP,YP
    1 FORMAT(6F10.5)
C     CLEAR ROUTINE FOR SYNTHESIS BY INVERSION(LINK 4 FIXED)
      DO 40 I=1,5
      DO 40 J=1,5
   40 T4(I,J)=0.
      Z=57.2957795131D0
      L3=DSQRT((XC-XP)**2+(YC-YP)**2)
      L4=DSQRT((XC-AX)**2+(YC-AY)**2)
      S3=DATAN2(YP-YC,XP-XC)*Z
      T4(1,1)=DATAN2(YC-AY,XC-AX)*Z
      IF(FNGN.EQ.1.) GO TO 201
    3 FORMAT(1H //14X,40HINPUT PARAMETER CONVERSION FOR SYNTHESIS/
     114X,43HOF SIX-LINK AND GEARED FIVE-LINK MECHANISMS/
     214X,42H(BASED ON STEPHENSON-2 SIX-LINK MECHANISM)//
```

```
     327X,13HCASE NUMBER  ,I1//11X,7HPOINT A,19X,7HPOINT B,19X,7HPOINT C
     4/ 4X,1H(,F9.4,1H,,F9.4,1H),5X,1H(,F9.4,1H,,F9.4,1H),5X,1H(,F9.4,1H
     5,,F9.4,1H)//21X,6HLINK 3,24X,6HLINK 4/17X,15HLENGTH    ANGLE,14X,
     615HLENGTH    ANGLE/15X,F9.4,1X,F9.4,10X,F9.4,1X,F9.4///24X,
     712HSTEPHENSON-2,17X,12HSTEPHENSON-3/ 2X,66HPOS. DER.   THETA 2
     8 THETA 3     THETA 5      PSI 3       PSI 4/)
      DO 20 I=1,5
      READ(NI,2)J,K,A1,A2,A3
    2 FORMAT(2I1,8X,3F10.5)
      ICASE=ICASE+J
      IC(I)=J
      IC(I+5)=K
      T2(J+1,K+1)=A1
      T3(J+1,K+1)=A2
   20 T5(J+1,K+1)=A3
      NC=JCASE(ICASE+1)
      WRITE(NO,3)NC,AX,AY,XC,YC,XP,YP,L3,S3,L4,T4(1,1)
      GO TO 202
  220 FORMAT(10X,33HINPUT PARAMETER CONVERSION - CASE,3X,I1/
     110X,31HFUNCTION GENERATION BY COGNATES/15X
     222H(FOUR-LINK MECHANISMS)//10X,7HPOINT A,15X,7HPOINT B,15X,
     37HPOINT C/3X,1H(,F9.4,1H,,F9.4,3H) (,F9.4,1H,,F9.4,3H) (,
     4F9.4,1H,,F9.4,1H)//20X,7HLINK AB,18X,7HLINK BC//16X
     5,15HLENGTH    ANGLE,8X,15HLENGTH    ANGLE/14X,F9.4,1X,F9.4
     6,4X,F9.4,1X,F9.4//2X,9HPOS. DER.,6X,5HTHETA,12X,3HPHI)
  201 DO 221 I=1,5
      READ(NI,222)J,K,A1,A2
  222 FORMAT(2I1,8X,2F10.5)
      ICASE=ICASE+J

      IC(I)=J
      IC(I+5)=K
      P4(J+1,K+1)=A1
  221 P3(J+1,K+1)=A2
```

```
      NC=JCASE(ICASE+1)
      WRITE(NO,220)NC,AX,AY,XC,YC,XP,YP,L4,T4(1,1),L3,S3
      DO 300 I=1,5
      M=IC(I)
      N=IC(I+5)
  300 WRITE(NO,223)M,N,P4(M+1,N+1),P3(M+1,N+1)
  223 FORMAT(3X,I1,4X,I1,2(4X,F12.5))
  202 NPT=JC(NC)
      IF(ENGN.EQ.1.)GO TO 203
      DO 30 I=1,NPT
      T5(I,1)=(T5(I,1)-T5(1,1))/Z
      T4(I,1)=T4(I,1)/Z
      T3(I,1)= T3(I,1)/Z
   30 T2(I,1)=T2(I,1)/Z
  203 DO 10 M=1,NPT
      IF(SIXINV.EQ.1.)GO TO 250
      P4(M,1)=P4(M,1)/Z
      P3(M,1)=P3(M,1)/Z
  250 IF(ENGN.EQ.1.) GO TO 205
      P2(M,1)=T2(M,1)-T5(M,1)
      P3(M,1)=T3(M,1)-T5(M,1)
      P4(M,1)=T4(1,1)-T5(M,1)
  205 A(M,1)=L4*DCOS(P4(M,1))+L3*DCOS(P3(M,1))+AX
   10 B(M,1)=L4*DSIN(P4(M,1))+L3*DSIN(P3(M,1))+AY
      IF(NC.EQ.7) GO TO 100
   12 NDER=1
      IF(NC.EQ.3.OR.NC.EQ.5) NDER=2
      DO 80 M=1,NDER
      IF(ENGN.EQ.1.) GO TO 206
      P2(M,2)=T2(M,2)-T5(M,2)
      P3(M,2)=T3(M,2)-T5(M,2)
      P4(M,2)=T4(M,2)-T5(M,2)
  206 A(M,2)=-P4(M,2)*L4*DSIN(P4(M,1))-P3(M,2)*L3*DSIN(P3(M,1))
   80 B(M,2)=P4(M,2)*L4*DCOS(P4(M,1))+P3(M,2)*L3*DCOS(P3(M,1))
```

```
      IF(NC.GT.4) GO TO 100
      S3=DSIN(P3(1,1))
      C3=DCOS(P3(1,1))
      S4=DSIN(P4(1,1))
      C4=DCOS(P4(1,1))
      IF(FNGN.EQ.1.) GO TO 207
      P2(1,3)=T2(1,3)-T5(1,3)
      P3(1,3)=T3(1,3)-T5(1,3)
      P4(1,3)=T4(1,3)-T5(1,3)
  207 A(1,3)=L4*(-P4(1,3)*S4-P4(1,2)*P4(1,2)*C4)+L3*(-P3(1,3)*S3-P3(1,2)
     1*P3(1,2)*C3)
      B(1,3)=L4*(-P4(1,2)*P4(1,2)*S4+P4(1,3)*C4)+L3*(-P3(1,2)*P3(1,2)*S3
     1+P3(1,3)*C3)
      IF(NC.GT.2) GO TO 100
      IF(FNGN.EQ.1.) GO TO 208
      P2(1,4)=T2(1,4)-T5(1,4)
      P3(1,4)=T3(1,4)-T5(1,4)
      P4(1,4)=T4(1,4)-T5(1,4)
  208 A(1,4)=L4*((-P4(1,4)+P4(1,2)**3)*S4-3.*P4(1,2)*P4(1,3)*C4)
     1+L3*((-P3(1,4)+P3(1,2)**3)*S3-3.*P3(1,2)*P3(1,3)*C3)
      B(1,4)=L4*(-3.*P4(1,2)*P4(1,3)*S4+(P4(1,4)-P4(1,2)**3)*C4)
     1+L3*(-3.*P3(1,2)*P3(1,3)*S3+(P3(1,4)-P3(1,2)**3)*C3)
      IF(NC.EQ.2) GO TO 100
      IF(FNGN.EQ.1.) GO TO 209
      P2(1,5)=T2(1,5)-T5(1,5)
      P3(1,5)=T3(1,5)-T5(1,5)
      P4(1,5)=T4(1,5)-T5(1,5)
  209 A(1,5)=L4*((-P4(1,5)+6.*P4(1,3)*P4(1,2)*P4(1,2))*S4
     1+(-4.*P4(1,2)*P4(1,4)-3.*P4(1,3)*P4(1,3)+P4(1,2)**4)*C4)
     2+L3*((-P3(1,5)+6.*P3(1,3)*P3(1,2)*P3(1,2))*S3
     3+(-4.*P3(1,2)*P3(1,4)-3.*P3(1,3)*P3(1,3)+P3(1,2)**4)*C3)
      B(1,5)=L4*((-3.*P4(1,3)*P4(1,3)-4.*P4(1,2)*P4(1,4)+P4(1,2)**4)*S4
     1+(-6.*P4(1,3)*P4(1,2)*P4(1,2)+P4(1,5))*C4)
     2+L3*((-3.*P3(1,3)*P3(1,3)-4.*P3(1,2)*P3(1,4)+P3(1,2)**4)*S3
```

```
     3+(-6.*P3(1,2)*P3(1,2)*P3(1,3)+P3(1,5))*C3)
 100 DO 50 I=1,NPT
     IF(FNGN.EQ.1.) GO TO 210
     T2(I,1)=T2(I,1)*Z
     T3(I,1)=T3(I,1)*Z
     T5(I,1)=T5(I,1)*Z
     P2(I,1)=P2(I,1)*Z
 210 P3(I,1)=P3(I,1)*Z
  50 P4(I,1)=P4(I,1)*Z
     IF(FNGN.EQ.1.) GO TO 211
     DO 60 I=1,5
     M=IC(I)+1
     N=IC(I+5)+1
  60 WRITE(NO,4)IC(I),IC(I+5),T2(M,N),T3(M,N),T5(M,N),P3(M,N),P4(M,N)
   4 FORMAT(1H ,2X,I1,4X,I1,3X,F9.4,3X,F9.4,3X,F9.4,3X,F9.4,3X,F9.4)
     WRITE(NO,6)
   6 FORMAT(1H //18X,38HCONVERTED SPECIFICATIONS FOR SYNTHESIS/
    12X,9HPOS. DER.,8X,2HXC,14X,2HYC,14X,5HPSI 2/)
     DO 70 I=1,5
     M=IC(I)+1
     N=IC(I+5)+1
  70 P(M,N)=P2(M,N)
     RETURN
 110 IEND=1
     RETURN
 211 DO 212 I=1,5
     M=IC(I)+1
     N=IC(I+5)+1
 212 P(M,N)=P3(M,N)
     WRITE(NO,5)
   5 FORMAT(//18X,38HCONVERTED SPECIFICATIONS FOR SYNTHESIS/
    1          2X,9HPOS. DER.,4X,11HX COMPONENT,5X,11HY COMPONENT,
    18X,4H PHI/)
     RETURN
```

```
COGS/                                   EXAMPLE 1
00          1.          1.              0.
01          2.          1.              3.
02          10.         6.              8.
10          1.5         2.              30.
11          3.          2.              4.
TRCE/CCGS/ANAL/                         EXAMPLE 2
00          1.          1.              0.
01          2.          6.              3.
02          10.         2.              8.
03           5.          8.              6.
10          1.5         2.              30.
FRFV/ANAL/COGS/                         EXAMPLE 3
00          1.          1.              0.
01          2.          1.              3.
02          10.         6.              8.
10          1.5         2.              30.
20          3.          2.              45.
ANAL/FGEN/CCGS/TRCE/                    EXAMPLE 4
1.          1.          2.              2.          4.          2.
00          45.         0.
01          2.          6.
02          4.          8.
10          60.         40.
11          2.          8.
CCGS/SIXL/TRCE/ANAL/                    EXAMPLE 5
1.          1.          2.              2.          4.          2.
00          0.          0.              0.
01          2.          4.              3.
10          20.         30.             15.
11          2.5         5.              4.
20          35.         40.             45.
ANAL/STLN/COGS/                         EXAMPLE 6
00          1.          1.              10.
```

```
01          4.          4.          2.
02          6.          6.          8.
03          5.          5.          6.
04          2.          2.          3.
FCUR/TEST/                          EXAMPLE 7                                   1.    181. 10.
00          1.          1.          0.
01          2.          6.          3.
02          10.         2.          8.
03          5.          8.          6.
CCGO/                               COGNATES, ETC. ONLY - EXAMPLE 8
1.          1.          2.          2.
0.          3.          3.          2.
3.          4.
FRFV/
00          1.          1.          0.
01          2.          1.          3.
02          10.         6.          8.
10          1.5         2.          30.
11          3.          2.          4.
FGEN/FRFV/
1.          1.          2.          2.          4.          2.
00          45.         0.
01          2.          6.
02          4.          8.
10          60.         40.
11          2.          8.
SIXL/FRFV/
1.          1.          2.          2.          4.          2.
00          0.          0.          0.
01          2.          4.          3.
10          20.         30.         15.
11          2.5         5.          4.
20          35.         40.         45.
```

The following pages illustrate portions of the output for these examples.

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**********************************************
*                                            *
*                  MECSYN                    *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS   *
**********************************************
```

OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	YES
INTERMEDIATE PRINTING	NO
ANALYSIS	NO
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	NO
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (45=BOTH)	5

EXAMPLE 1

SPECIFICATIONS - CASE 3

POS.	DER.	X COMPONENT	Y COMPONENT	THETA
0	0	1.00000	1.00000	0.0
0	1	2.00000	1.00000	3.00000
0	2	10.00000	6.00000	8.00000
1	0	1.50000	2.00000	30.00000
1	1	3.00000	2.00000	4.00000

COORDINATES OF THE CONSTRAINT LINKS

POINTS				CENTERS			
X =	1.69751	Y =	1.39767	X =	-4.90641	Y =	3.12092
X =	0.09736	Y =	1.32287	X =	0.40243	Y =	1.50710
X =	0.95101	Y =	1.86623	X =	1.50082	Y =	2.25211
X =	0.65653	Y =	1.63390	X =	0.77148	Y =	2.00544

ORIGINAL MECHANISM
A(1.698, 1.398)
OA(-4.906, 3.121)
B(0.097, 1.323)
OB(0.402, 1.507)

COUPLER POINT
(1.000, 1.000)
FIRST COGNATE
O(-5.604, 2.723)
OO(-4.906, 3.121)
E(-2.263, 3.457)
OE(-2.168, 3.609)

SECOND COGNATE
F(1.305, 1.184)
OF(0.402, 1.507)
G(1.094, 1.152)
OG(-2.168, 3.609)

DOUBLE-ROCKER
LINK LENGTH RATIO 19.15
MODIFIED L.L.R. 19.15
ORIGINAL MECHANISM
A(1.698, 1.398)
OA(-4.906, 3.121)
B(0.951, 1.866)
OB(1.501, 2.252)

DOUBLE-ROCKER
LINK LENGTH RATIO 19.15
MODIFIED L.L.R. 30.54
FIRST COGNATE
O(-5.604, 2.723)
OO(-4.906, 3.121)
E(-1.378, 7.284)
OE(-1.451, 7.891)

DOUBLE-ROCKER
LINK LENGTH RATIO 19.15
MODIFIED L.L.R. 19.15
SECOND COGNATE
F(1.550, 1.386)
OF(1.501, 2.252)
G(0.927, 1.608)
OG(-1.451, 7.891)

DOUBLE-ROCKER
LINK LENGTH RATIO 10.16
MODIFIED L.L.R. 10.16
ORIGINAL MECHANISM
A(0.097, 1.323)
OA(0.402, 1.507)
B(0.951, 1.866)
OB(1.501, 2.252)

DOUBLE-ROCKER
LINK LENGTH RATIO 10.16
MODIFIED L.L.R. 11.07
FIRST COGNATE
O(1.305, 1.184)
OO(0.402, 1.507)
E(0.990, 1.305)
OE(1.598, 1.118)

DOUBLE-ROCKER
LINK LENGTH RATIO 10.16
MODIFIED L.L.R. 10.47
SECOND COGNATE
F(1.550, 1.386)
OF(1.501, 2.252)
G(1.608, 0.813)
OG(1.598, 1.118)

CRANK-ROCKER
LINK LENGTH RATIO 3.72
MODIFIED L.L.R. 3.72
ORIGINAL MECHANISM
A(1.698, 1.398)
OA(-4.906, 3.121)
B(0.657, 1.634)
OB(0.771, 2.005)

DOUBLE-ROCKER
LINK LENGTH RATIO 3.72
MODIFIED L.L.R. 3.80
FIRST COGNATE
O(-5.604, 2.723)
OO(-4.906, 3.121)
E(-1.065, 5.121)
OE(-1.190, 5.386)

CRANK-ROCKER
LINK LENGTH RATIO 3.72
MODIFIED L.L.R. 3.72
SECOND COGNATE
F(1.115, 1.372)
OF(0.771, 2.005)
G(0.875, 1.265)
OG(-1.190, 5.386)

DOUBLE-ROCKER
LINK LENGTH RATIO 17.55
MODIFIED L.L.R. 17.55
ORIGINAL MECHANISM
A(0.097, 1.323)
OA(0.402, 1.507)
B(0.657, 1.634)
OB(0.771, 2.005)

DOUBLE-ROCKER
LINK LENGTH RATIO 17.55
MODIFIED L.L.R. 19.54
FIRST COGNATE
O(1.305, 1.184)
OO(0.402, 1.507)
E(0.796, 1.346)
OE(1.328, 1.583)

DOUBLE-ROCKER
LINK LENGTH RATIO 17.55
MODIFIED L.L.R. 17.55
SECOND COGNATE
F(1.115, 1.372)
OF(0.771, 2.005)
G(1.532, 1.237)
OG(1.328, 1.583)

CRANK-ROCKER
LINK LENGTH RATIO 1.80

DOUBLE-ROCKER
LINK LENGTH RATIO 1.80

CRANK-ROCKER
LINK LENGTH RATIO 1.80

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**************************************************
*                                                *
*                    MECSYN                      *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
**************************************************
```

OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	YES
INTERMEDIATE PRINTING	YES
ANALYSIS	YES
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	NO
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (45=BOTH)	5

EXAMPLE 2

SPECIFICATIONS - CASE 2

POS.	DER.	X COMPONENT	Y COMPONENT	THETA
0	0	1.00000	1.00000	0.0
0	1	2.00000	6.00000	3.00000
0	2	10.00000	2.00000	8.00000
0	3	5.00000	8.00000	6.00000
1	0	1.50000	2.00000	30.00000

RIGHT MATRIX DEFINED BY Z0,Z1 AND Z2

-2.125000	-1.299038.	0.017949
-8.000000	-5.000000	-3.000000
-52.000000	-45.000000	27.000000
-109.000000	-20.000000	343.000000

LEFT MATRIX DEFINED BY Z3,Z4,Z5 AND Z6

-0.133975	0.500000	0.500000	1.000000
0.0	3.000000	2.000000	6.000000
-9.000000	8.000000	10.000000	2.000000
-72.000000	-21.000000	5.000000	8.000000

COEFFICIENTS DEFINING Z3 THRU Z6 IN TERMS OF Z0,Z1 AND Z2

1.392659	0.906749	-4.454070
-0.416372	-2.270869	-1.096397
-3.630515	-2.065341	-0.472698
0.085024	0.990548	0.205765

THE QUARTIC DEFINING X IN THE MOVING PLANE

$$0.23471(X)^4 + 6.57242(X)^3 + 31.01007(X)^2 + 43.69103(X) + 10.49217 = 0$$

COORDINATES OF THE CONSTRAINT LINKS

POINTS				CENTERS			
X =	-1.20331	Y =	1.40838	X =	-0.72704	Y =	2.01343
X =	-2.00672	Y =	2.90347	X =	-1.67962	Y =	2.50161
X =	0.69999	Y =	1.04946	X =	3.03428	Y =	0.20197
X =	-21.49206	Y =	88.18033	X =	-1.61328	Y =	4.25581

ORIGINAL MECHANISM			COUPLER POINT (1.000, 1.000) FIRST COGNATE			SECOND COGNATE		
A(	-1.203.	1.408)	D(	1.476.	1.605)	F(	1.327.	0.598)
OA(	-0.727.	2.013)	OD(	-0.727.	2.013)	OF(	-1.680.	2.502)
B(	-2.007.	2.903)	E(	1.247.	2.595)	G(	0.316.	0.995)
OB(	-1.680.	2.502)	OE(	0.563.	2.591)	OG(	0.563.	2.591)

DOUBLE-ROCKER		DOUBLE-ROCKER		DOUBLE-ROCKER	
LINK LENGTH RATIO	3.28	LINK LENGTH RATIO	3.28	LINK LENGTH RATIO	3.28
MODIFIED L.L.R.	5.60	MODIFIED L.L.R.	3.28	MODIFIED L.L.R.	5.92

ANALYSIS OF ANGULAR PROPERTIES

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	67.39915	231.79167	349.49955	309.69128	333.80581	327.66326
0	1	4.89889	1.28065	3.00000	3.17955	1.04464	3.00000
0	2	-27.73696	42.61041	8.00000	6.87346	-0.55129	8.00000
0	3	-307.68170	157.86281	6.00000	-155.81889	-54.99062	6.00000
1	0	80.74041	278.56836	19.49955	339.80922	357.66326	344.40687
		LINK LENGTH	0.77002	LINK LENGTH	2.24084	LINK LENGTH	3.55859

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	129.69128	169.49955	51.79167	247.39915	352.32456	309.14451
0	1	3.17955	3.00000	1.28065	4.89889	-5.18842	9.23306
0	2	6.87346	8.00000	42.61041	-27.73696	-7.61257	15.06127
0	3	-155.81889	6.00000	157.86281	-307.68170	-524.97434	476.07372
1	0	159.80922	199.49955	98.56836	260.74041	43.47533	298.59480
		LINK LENGTH	2.24084	LINK LENGTH	0.77002	LINK LENGTH	0.51816

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	129.14451	172.32456	0.39219	210.34129	309.53041	261.20306
0	1	9.23306	-5.18842	9.23306	-4.96763	2.98478	1.28065
0	2	15.06127	-7.61257	15.06127	24.44683	-3.10231	42.61041
0	3	476.07372	-524.97434	476.07372	-134.48492	183.72599	157.86281
1	0	118.59480	223.47533	94.72300	200.82377	347.56703	307.97975
		LINK LENGTH	0.51816	LINK LENGTH	0.68410	LINK LENGTH	1.61446

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	147.66326	153.80581	81.20306	129.53041	30.34129	180.39219
0	1	3.00000	1.04464	1.28065	2.98478	-4.96763	9.23306

```
*************************************************
*                                               *
*                    MECSYN                     *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
*************************************************
```

OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	YES
INTERMEDIATE PRINTING	NO
ANALYSIS	YES
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	NO
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (45=BOTH)	45

EXAMPLE 3

SPECIFICATIONS - CASE 2

POS.	DER.	X COMPONENT	Y COMPONENT	THETA
0	0	1.00000	1.00000	0.0
0	1	2.00000	1.00000	3.00000
0	2	10.00000	6.00000	8.00000
1	0	1.50000	2.00000	30.00000
2	0	3.00000	2.00000	45.00000

FOUR MULTIPLY SEPARATED POSITION
KINEMATIC SYNTHESIS

FOCAL COORDINATES (0.61202D 00, 0.14245D 01)

START	STOP	STEP	NPOS
1	171	10	45

CIRCLE POINTS		CENTER POINTS	
1.6725	1.4245	-5.9196	3.2523
0.0188	1.4245	0.3421	1.5454
1.5148	1.5837	31.1127	-1.3118
0.0887	1.3322	0.3958	1.5099
1.3742	1.7019	5.3015	1.8976
0.1661	1.2622	0.4537	1.4946
1.2402	1.7872	3.0406	2.1655
0.2497	1.2153	0.5125	1.4998
1.1095	1.8419	2.1290	2.2454
0.3370	1.1937	0.5686	1.5260
0.9821	1.8655	1.6022	2.2565
0.4242	1.2007	0.6185	1.5740
0.8600	1.8540	1.2418	2.2240
0.5068	1.2423	0.6590	1.6462
0.7462	1.7932	0.9698	2.1488
0.5793	1.3347	0.6890	1.7514
-2.6565	4.1671	0.0979	2.0947
-0.7577	2.5738	0.1796	1.9769
3.3550	-0.1592	-0.3179	2.3354
-0.2051	1.8963	0.2313	1.7813
2.2276	0.8365	-0.9201	2.5106
-0.1093	1.6870	0.2590	1.6774
1.8768	1.2015	-2.0950	2.7283
-0.0449	1.5403	0.2958	1.6008

FIVE POSITION SOLUTION - CASE 4

COORDINATES OF THE CONSTRAINT LINKS

POINTS			CENTERS		
X = 1.00737	Y = 1.86341		X = 1.69095	Y = 2.25815	
X = -0.50694	Y = 2.31641		X = 0.19897	Y = 1.92560	

COUPLER POINT
(1.000, 1.000)

ORIGINAL MECHANISM		FIRST COGNATE		SECOND COGNATE	
A(	1.007, 1.863)	D(	1.684, 1.395)	F(	1.706, 0.609)
OA(	1.691, 2.258)	OO(	1.691, 2.258)	OF(	0.199, 1.926)
B(	-0.507, 2.316)	E(	1.995, 1.096)	G(	1.098, 1.430)
OB(	0.199, 1.926)	OE(	2.092, 1.526)	OG(	2.092, 1.526)

DOUBLE-ROCKER		DOUBLE-ROCKER		DOUBLE-ROCKER	
LINK LENGTH RATIO	2.00	LINK LENGTH RATIO	2.00	LINK LENGTH RATIO	2.00
MODIFIED L.L.R.	2.00	MODIFIED L.L.R.	2.07	MODIFIED L.L.R.	3.21

ANALYSIS OF ANGULAR PROPERTIES

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	272.44573	210.00483	269.51089	212.93966	318.86055	302.88634
0	1	3.26710	-1.49523	3.00000	-1.22813	3.00000	0.54013
0	2	8.93435	3.79509	8.00000	4.72945	8.00000	7.57338
1	0	325.69401	141.32622	299.51089	167.50933	17.68547	348.86055
2	0	26.93600	310.75198	23.17709	314.51089	3.86055	359.18255
		LINK LENGTH	0.78937	LINK LENGTH	0.86344	LINK LENGTH	2.00095

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	32.93966	89.51089	30.00483	92.44573	331.02961	290.71729
0	1	-1.22813	3.00000	-1.49523	3.26710	4.98762	-1.44749
0	2	4.72945	8.00000	3.79509	8.93435	11.58950	3.98388
1	0	347.50933	119.51089	321.32622	145.69401	41.38812	325.15791
2	0	134.51089	203.17709	130.75198	206.93600	175.71278	187.33033
		LINK LENGTH	0.86344	LINK LENGTH	0.78937	LINK LENGTH	0.80687

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	110.71729	151.02961	257.19498	154.22379	225.89891	185.51986
0	1	-1.44749	4.98762	4.98762	-5.04271	1.44014	-1.49523
0	2	3.98388	11.58950	11.58950	-13.55448	-5.76007	3.79509
1	0	145.15791	221.38812	251.32328	31.33464	165.81667	116.84125
2	0	7.33033	355.71278	101.87815	313.27812	52.70523	2.45104
		LINK LENGTH	0.80687	LINK LENGTH	0.44077	LINK LENGTH	0.99929

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	122.88634	138.86055	5.51986	45.89891	334.22379	77.19498
0	1	0.54013	3.00000	-1.49523	1.44014	-5.04271	4.98762

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**************************************************
*                                                *
*                    MECSYN                      *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*    FOUR AND FIVE MULTIPLY SEPARATED POSITIONS   *
**************************************************
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OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	YES
INTERMEDIATE PRINTING	YES
ANALYSIS	YES
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	YES
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (45=BOTH)	5

EXAMPLE 4

INPUT PARAMETER CONVERSION - CASE 3
FUNCTION GENERATION BY COGNATES
(FOUR-LINK MECHANISMS)

POINT A	POINT B	POINT C
(1.0000, 1.0000)	(2.0000, 2.0000)	(4.0000, 2.0000)

LINK AB		LINK BC	
LENGTH	ANGLE	LENGTH	ANGLE
1.4142	45.0000	2.0000	0.0

POS.	DER.	THETA	PHI
0	0	45.00000	0.0
0	1	2.00000	6.00000
0	2	4.00000	8.00000
1	0	60.00000	40.00000
1	1	2.00000	8.00000

CONVERTED SPECIFICATIONS FOR SYNTHESIS

POS.	DER.	X COMPONENT	Y COMPONENT	PHI
0	0	4.00000	2.00000	0.0
0	1	-2.00000	14.00000	6.00000
0	2	-80.00000	16.00000	8.00000
1	0	3.23920	3.51032	40.00000
1	1	-12.73409	13.67092	8.00000

RIGHT MATRIX DEFINED BY Z0, Z1 AND Z2

-1.407368	-0.737758	1.393054
-20.000000	-10.000000	10.000000
-6.741107	-3.888192	19.244213
88.000000	40.000000	64.000000

LEFT MATRIX DEFINED BY Z3, Z4, Z5 AND Z6

-0.233956	0.642788	-0.760804	1.510320
0.0	6.000000	-2.000000	14.000000
-5.142301	6.128356	-12.734091	13.670925
-36.000000	8.000000	-80.000000	16.000000

COEFFICIENTS DEFINING Z3 THRU Z6 IN TERMS OF Z0, Z1 AND Z2

-176.094405	-89.047202	105.942662
-258.675377	-130.337689	161.289976
76.342623	39.671312	-47.380939
120.338394	60.669197	-75.178695

THE QUARTIC DEFINING X IN THE MOVING PLANE

$$**********(X)^4 + **********(X)^3 + **********(X)^2 + **********(X) + ********** = 0$$

COORDINATES OF THE CONSTRAINT LINKS

	POINTS				CENTERS		
X =	2.00000	Y =	2.00000	X =	1.00000	Y =	1.00000
X =	1.57443	Y =	1.66542	X =	1.60472	Y =	1.66583
X =	1.81962	Y =	1.92433	X =	4.39017	Y =	6.25466
X =	1.64324	Y =	1.72212	X =	1.63001	Y =	1.75344

ORIGINAL MECHANISM
A(2.000, 2.000)
OA(1.000, 1.000)
B(1.574, 1.665)
OB(1.605, 1.666)

COUPLER POINT
(4.000, 2.000)

FIRST COGNATE
D(3.000, 1.000)
OD(1.000, 1.000)
E(-2.188, 0.379)
OE(-2.277, 0.447)

SECOND COGNATE
F(4.030, 2.000)
OF(1.605, 1.666)
G(3.911, 2.068)
OG(-2.277, 0.447)

DOUBLE-ROCKER
LINK LENGTH RATIO 46.69
MODIFIED L.L.P. 73.43

DOUBLE-ROCKER
LINK LENGTH RATIO 46.69
MODIFIED L.L.R. 46.69

DOUBLE-ROCKER
LINK LENGTH RATIO 46.69
MODIFIED L.L.R. 89.96

ANALYSIS OF ANGULAR PROPERTIES

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	45.00000	351.86990	36.86990	0.00000	8.03063	7.85375
0	1	2.00000	6.80000	2.80000	6.00000	5.69499	6.00000
0	2	4.00000	7.52000	3.52000	8.00000	14.69422	8.00007
1	0	60.00000	36.53433	56.53433	40.00000	49.05531	47.85375
1	1	2.00000	9.06047	3.06047	8.00000	7.09228	8.00000
		LINK LENGTH	1.41421	LINK LENGTH	2.00000	LINK LENGTH	2.44854

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	180.00000	216.86990	171.86990	225.00000	15.10971	0.77467
0	1	6.00000	2.80000	6.80000	2.00000	-6.57677	18.27176
0	2	8.00000	3.52000	7.52000	4.00000	303.43475	-280.74045
1	0	220.00000	236.53433	216.53433	240.00000	170.50825	286.40081
1	1	8.00000	3.06047	9.06047	2.00000	76.66833	-61.57605
		LINK LENGTH	2.00000	LINK LENGTH	1.41421	LINK LENGTH	0.03029

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	180.77467	195.10971	65.19415	322.60064	14.67972	13.11507
0	1	18.27176	-6.57677	-13.91954	18.27174	2.00000	2.35220
0	2	-280.74045	303.43475	289.58067	-280.74628	4.00000	4.83440
1	0	106.40081	350.50825	169.86604	248.22676	29.67971	28.41308
1	1	-61.57605	76.66833	67.32435	-61.57609	2.00000	3.74827
		LINK LENGTH	0.03029	LINK LENGTH	0.11191	LINK LENGTH	6.39656

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	187.85375	188.03063	193.11507	194.67972	142.60064	245.19415
0	1	6.00000	5.69499	2.35220	2.00000	18.27174	-13.91954

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**************************************************
*                      MECSYN                    *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
**************************************************

              OPTIONS
COGNATES, GRASHOF CRITERION
AND LINK LENGTH RATIOS              YES
INTERMEDIATE PRINTING               YES
ANALYSIS                            YES
STRAIGHT LINE SPECIFICATIONS         NO
FUNCTION GENERATION(FOUR-LINK)       NO
FIVE- AND SIX-LINK SYNTHESIS        YES
FOUR OR FIVE MSP (45=BOTH)            5

EXAMPLE 5


    INPUT PARAMETER CONVERSION FOR SYNTHESIS
    OF SIX-LINK AND GEARED FIVE-LINK MECHANISMS
    (BASED ON STEPHENSON-2 SIX-LINK MECHANISM)

                CASE NUMBER  5

      POINT A                 POINT B                 POINT C
(   1.0000,   1.0000)   (   2.0000,   2.0000)   (   4.0000,   2.0000)

            LINK 3                  LINK 4
        LENGTH    ANGLE         LENGTH    ANGLE
        2.0000     0.0          1.4142   45.0000

                    STEPHENSON-2                STEPHENSON-3
POS.  DER.   THETA 2    THETA 3    THETA 5    PSI 3      PSI 4

 0     0      0.0        0.0        0.0        0.0      45.0000
 0     1      2.0000     4.0000     3.0000     1.0000   -3.0000
 1     0     20.0000    30.0000    15.0000    15.0000   30.0000
 1     1      2.5000     5.0000     4.0000     1.0000   -4.0000
 2     0     35.0000    40.0000    45.0000    -5.0000    0.0000

          CONVERTED SPECIFICATIONS FOR SYNTHESIS
POS.  DER.      XC          YC          PSI 2
```

0	0	4.00000	2.00000	0.0
0	1	3.00000	-1.00000	-1.00000
1	0	4.15660	2.22474	5.00000
1	1	2.31079	-2.96713	-1.50000
2	0	4.40660	0.82569	-10.00000

RIGHT MATRIX DEFINED BY Z0,Z1 AND Z2

-1.113392	-0.334679	0.145992
-0.049956	-0.196277	0.421657
-10.000000	-1.000000	-3.000000
-3.003915	0.737618	-3.344783

LEFT MATRIX DEFINED BY Z3,Z4,Z5 AND Z6

-0.003805	0.087156	0.156597	0.224745
-0.015192	-0.173648	0.406603	-1.174311
0.0	-1.000000	3.000000	-1.000000
0.130734	-1.494292	2.310789	-2.967128

COEFFICIENTS DEFINING Z3 THRU Z6 IN TERMS OF Z0,Z1 AND Z2

5.866322	1.202933	-1.445843
-1.527143	-2.238697	3.963438
-4.271188	-1.038558	0.013940
-1.286421	0.123023	-0.921619

THE QUARTIC DEFINING X IN THE MOVING PLANE

$$0.03099(X)^4 + 0.40034(X)^3 + 1.89231(X)^2 + 3.82686(X) + 2.79432 = 0$$

COORDINATES OF THE CONSTRAINT LINKS

POINTS				CENTERS			
X =	1.95051	Y =	0.89544	X =	2.15807	Y =	0.52057
X =	1.67383	Y =	3.08038	X =	1.84026	Y =	2.56829

ORIGINAL MECHANISM
A(1.951, 0.895)
OA(2.158, 0.521)
B(1.674, 3.080)
OB(1.840, 2.568)

COUPLER POINT
(4.000, 2.000)

FIRST COGNATE
O(4.208, 1.625)
OO(2.158, 0.521)
E(4.498, 1.973)
OE(4.057, 1.613)

SECOND COGNATE
F(4.166, 1.488)
OF(1.840, 2.568)
G(3.558, 1.641)
OG(4.057, 1.613)

DOUBLE-ROCKER
LINK LENGTH RATIO 5.14
MODIFIED L.L.R. 5.71

DOUBLE-ROCKER
LINK LENGTH RATIO 5.14
MODIFIED L.L.R. 5.14

DOUBLE-ROCKER
LINK LENGTH RATIO 5.14
MODIFIED L.L.R. 5.14

ANALYSIS OF ANGULAR PROPERTIES

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	118.97292	318.56968	49.22039	28.32222	355.42815	335.08768
0	1	-5.05628	2.80588	-1.25041	-1.00000	0.81760	-1.00000
1	0	82.88106	358.02849	47.58733	33.32222	3.03966	340.08768
1	1	-0.92268	-1.93827	-1.36095	-1.50000	-0.71723	-1.50000
2	0	100.31737	275.13791	18.32222	357.13305	326.55750	325.08768
		LINK LENGTH	0.42849	LINK LENGTH	2.32819	LINK LENGTH	2.56482

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	208.32222	229.22039	138.56968	298.97292	42.51107	288.00476
0	1	-1.00000	-1.25041	2.80588	-5.05628	7.78571	-7.96811
1	0	213.32222	227.58733	178.02849	262.88106	62.94946	280.17788
1	1	-1.50000	-1.36095	-1.93827	-0.92268	4.61523	-6.83245
2	0	177.13305	198.32222	95.13791	280.31737	142.31991	149.32527
		LINK LENGTH	2.32819	LINK LENGTH	0.42849	LINK LENGTH	0.53846

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	108.00476	222.51107	157.54462	39.10994	176.84356	19.81100
0	1	-7.96811	7.78571	-6.48712	-7.96811	-5.05628	-9.39895
1	0	100.17788	242.94946	130.13223	31.28306	140.75169	20.66359
1	1	-6.83245	4.61523	-2.07781	-6.83245	-0.92268	-7.98758
2	0	329.32527	322.31991	327.48296	260.43045	333.00854	254.90486
		LINK LENGTH	0.53846	LINK LENGTH	0.56921	LINK LENGTH	0.49901

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	155.08768	175.42815	199.81100	356.84356	219.10994	337.54462
0	1	-1.00000	0.81760	-9.39895	-5.05628	-7.96811	-6.48712

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*                    MECSYN                      *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
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OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	YES
INTERMEDIATE PRINTING	NO
ANALYSIS	YES
STRAIGHT LINE SPECIFICATIONS	YES
FUNCTION GENERATION(FOUR-LINK)	NO
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (AS=BOTH)	5

EXAMPLE 6

SPECIFICATIONS - CASE 1

POS.	DER.	X COMPONENT	Y COMPONENT	THETA
0	0	1.00000	1.00000	10.00000
0	1	4.00000	4.00000	2.00000
0	2	6.00000	6.00000	8.00000
0	3	5.00000	5.00000	6.00000
0	4	2.00000	2.00000	3.00000

COORDINATES OF THE CONSTRAINT LINKS

POINTS				CENTERS			
X =	1.42411	Y =	1.06345	X =	-9.56193	Y =	9.83986
X =	-0.30044	Y =	4.19368	X =	-0.47582	Y =	3.89442
X =	0.42355	Y =	2.87957	X =	-0.89064	Y =	2.99075

COUPLER POINT
(1.000, 1.000)

ORIGINAL MECHANISM		FIRST COGNATE		SECOND COGNATE	
A(	1.424, 1.063)	D(	-9.986, 9.776)	F(	0.825, 0.701)
OA(	-9.562, 9.840)	OD(	-9.562, 9.840)	OF(	-0.476, 3.894)
B(	-0.300, 4.194)	E(	-8.540, 10.646)	G(	1.026, 0.968)
OB(	-0.476, 3.894)	OE(	-8.514, 10.614)	OG(	-8.514, 10.614)

CRANK-ROCKER		CRANK-ROCKER		DOUBLE-ROCKER	
LINK LENGTH RATIO	40.54	LINK LENGTH RATIO	40.54	LINK LENGTH RATIO	40.54
MODIFIED L.L.R.	40.54	MODIFIED L.L.R.	331.91	MODIFIED L.L.R.	69.85

ANALYSIS OF ANGULAR PRORERTIES

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	321.37973	318.76464	91.63558	188.50879	301.87692	292.15580
0	1	0.44131	0.37691	-1.18178	2.00000	1.31213	2.00000
0	2	0.67630	0.49161	-6.83209	8.00000	0.52917	8.00000
0	3	-0.52865	1.01661	-5.51204	6.00000	1.73993	6.00000
0	4	-7.93808	5.36061	-5.57747	3.00000	-176.16950	3.00000
		LINK LENGTH	14.06125	LINK LENGTH	0.42883	LINK LENGTH	3.44829

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	8.50879	271.63558	138.76464	141.37973	354.40524	239.62748
0	1	2.00000	-1.18178	0.37691	0.44131	-4.66550	7.97763
0	2	8.00000	-6.83209	0.49161	0.67630	-2.11734	10.64651
0	3	6.00000	-5.51204	1.01661	-0.52865	-104.00963	111.74956
0	4	3.00000	-5.57747	5.36061	-7.93808	-1326.08046	1152.91097
		LINK LENGTH	0.42883	LINK LENGTH	14.06125	LINK LENGTH	0.34686

POS.	DER.	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2	CONSTRAINT LINK CLOSURE 1	CONSTRAINT LINK CLOSURE 2
0	0	59.62748	174.40524	129.28429	140.11600	314.71673	314.68355
0	1	7.97763	-4.66550	7.97763	-7.14119	0.39513	0.44131
0	2	10.64651	-2.11734	10.64651	-9.38819	0.58202	0.67630
0	3	111.74956	-104.00963	111.74947	-110.98004	1.29810	-0.52865
0	4	1152.91097	-1326.08046	1152.88188	-1155.06650	5.75845	-7.93810
		LINK LENGTH	0.34686	LINK LENGTH	0.04162	LINK LENGTH	13.56724

POS.	DER.	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2	COUPLER LINK CLOSURE 1	COUPLER LINK CLOSURE 2
0	0	112.15580	121.87692	134.68355	134.71673	320.11600	309.28429
0	1	2.00000	1.31213	0.44131	0.39513	-7.14119	7.97763

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*                    MECSYN                      *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
**************************************************
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OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	NO
INTERMEDIATE PRINTING	NO
ANALYSIS	NO
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	NO
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (45=BOTH)	4

EXAMPLE 7

SPECIFICATIONS - CASE 1

POS.	DER.	X COMPONENT	Y COMPONENT	THETA
0	0	1.00000	1.00000	0.0
0	1	2.00000	6.00000	3.00000
0	2	10.00000	2.00000	8.00000
0	3	5.00000	8.00000	6.00000

FOUR MULTIPLY SEPARATED POSITION
KINEMATIC SYNTHESIS

FOCAL COORDINATES (0.12457D 01, 0.14779D 01)

START	STOP	STEP	NPOS
1	181	10	4

CIRCLE POINTS		CENTER POINTS	
-1.1530	1.4779	-0.7893	1.9267
-0.7671	1.4779	-0.7443	1.4594
-1.4491	1.0029	-0.4733	2.4469
-0.3132	1.2030	-0.0858	1.0495
-1.6964	0.4060	-0.2906	2.0291
0.0651	1.0591	0.7713	0.6840
-1.8913	-0.3333	-0.2445	3.3619
0.4519	1.0196	1.8804	0.3829
-2.0056	-1.2503	-0.2870	3.7348
0.7512	1.0629	3.3401	0.1704
-2.0187	-2.4125	-0.4077	4.0384
0.9876	1.1703	5.3508	0.0807
-1.8932	-3.9598	-0.5874	4.2653
1.1570	1.3242	8.3842	0.1767
-1.6541	-6.2146	-0.8071	4.4102
1.2567	1.5080	13.8340	0.6237
-0.7988	-10.1171	-1.0479	4.4700
1.2859	1.7057	28.2292	2.1658
1.2457	-19.7783	-1.2909	4.4441
1.2457	1.9018	331.6846	36.5058
23.3026	-123.6127	-1.5176	4.3348
1.1393	2.0817	-34.7384	-4.8782
-9.0295	29.7089	-1.7100	4.1464
0.9716	2.2312	-15.7842	-2.5666
-5.2249	12.6855	-1.8510	3.8861
0.7495	2.3375	-9.5687	-1.6188
-3.8908	7.5995	-1.9241	3.5632
0.4816	2.3886	-6.3491	-0.9398
-3.0514	5.0836	-1.9138	3.1888
0.1780	2.3739	-4.3367	-0.3365
-2.3846	3.5739	-1.8055	2.7762
-0.1500	2.2838	-2.9593	0.2443
-1.7985	2.5860	-1.5847	2.3398
-0.4900	2.1097	-1.9786	0.8167
-1.2623	1.9202	-1.2366	1.8953
-0.8289	1.8437	-1.2772	1.3799

```
*****************************************************
*                      MECSYN                       *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
*****************************************************
```

OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	YES
INTERMEDIATE PRINTING	NO
ANALYSIS	NO
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	NO
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (45=BOTH)	0

EXAMPLE 8

COUPLER POINT
(3.000, 4.000)

ORIGINAL MECHANISM		FIRST COGNATE		SECOND COGNATE	
A(	2.000, 2.000)	D(	2.000, 3.000)	F(	0.000, 5.000)
OA(	1.000, 1.000)	OD(	1.000, 1.000)	OF(	0.0 , 3.000)
B(	3.000, 2.000)	E(	1.000, 0.000)	G(	-2.000, -1.000)
OB(	0.0 , 3.000)	OE(	-4.000, 1.000)	OG(	-4.000, 1.000)

DOUBLE-ROCKER		DOUBLE-ROCKER		DOUBLE-ROCKER	
LINK LENGTH RATIO	3.16	LINK LENGTH RATIO	3.16	LINK LENGTH RATIO	3.16
MODIFIED L.L.R.	3.16	MODIFIED L.L.R.	3.33	MODIFIED L.L.R.	3.16

```
**************************************************
*                    MECSYN                      *
*MODULAR SYNTHESIS OF MULTI-LINK PLANAR MECHANISMS*
* FOR TIME DEPENDENT AND GEOMETRIC SPECIFICATIONS *
*   FOUR AND FIVE MULTIPLY SEPARATED POSITIONS    *
**************************************************
```

OPTIONS

COGNATES, GRASHOF CRITERION AND LINK LENGTH RATIOS	NO
INTERMEDIATE PRINTING	NO
ANALYSIS	NO
STRAIGHT LINE SPECIFICATIONS	NO
FUNCTION GENERATION(FOUR-LINK)	YES
FIVE- AND SIX-LINK SYNTHESIS	NO
FOUR OR FIVE MSP (45=BOTH)	45

INPUT PARAMETER CONVERSION - CASE 3
FUNCTION GENERATION BY COGNATES
(FOUR-LINK MECHANISMS)

POINT A	POINT B	POINT C
(1.0000, 1.0000)	(2.0000, 2.0000)	(4.0000, 2.0000)

LINK AB		LINK BC	
LENGTH	ANGLE	LENGTH	ANGLE
1.4142	45.0000	2.0000	0.0

POS.	DER.	THETA	PHI
0	0	45.00000	0.0
0	1	2.00000	6.00000
0	2	4.00000	8.00000
1	0	60.00000	40.00000
1	1	2.00000	8.00000

CONVERTED SPECIFICATIONS FOR SYNTHESIS

POS.	DER.	X COMPONENT	Y COMPONENT	PHI
0	0	4.00000	2.00000	0.0
0	1	-2.00000	14.00000	6.00000
0	2	-80.00000	16.00000	8.00000
1	0	3.23920	3.51032	40.00000
1	1	-12.73409	13.67092	8.00000

FOUR MULTIPLY SEPARATED POSITION
KINEMATIC SYNTHESIS

FOCAL COORDINATES (0.23043D 01, 0.17212D 01)

START	STOP	STEP	NPOS
1	171	10	45

CIRCLE POINTS		CENTER POINTS	
1.6430	1.7212	1.6295	1.7522
1.5306	1.7212	1.6204	1.6852
1.7505	1.6236	1.5863	1.7080
1.5556	1.5892	1.5901	1.6133
1.9100	1.5777	1.5481	1.7100
1.5127	1.4331	1.5847	1.5423
2.0431	1.5704	1.5081	1.7072
1.4892	1.2506	1.5943	1.4971
2.1494	1.5912	1.4671	1.6979
1.4924	1.0399	1.6117	1.4691
2.2296	1.6321	1.4259	1.6815
1.5288	0.7970	1.6330	1.4540
2.2842	1.6863	1.3854	1.6577
1.6068	0.5130	1.6554	1.4491
2.3141	1.7480	1.3462	1.6264
1.7396	0.1695	1.6771	1.4527
2.3203	1.8118	1.3086	1.5872
1.9525	-0.2740	1.6966	1.4634
2.3043	1.8727	1.2727	1.5394
2.3043	-0.9168	1.7128	1.4799
2.2692	1.9263	1.2376	1.4815
2.9718	-2.0642	1.7246	1.5011
2.2143	1.9686	1.2011	1.4100
4.8445	-5.2578	1.7314	1.5256
2.1456	1.9961	1.1577	1.3165
-4459.0542	7729.0251	1.7326	1.5524
2.0655	2.0059	1.0922	1.1780
-2.0542	6.9155	1.7280	1.5801
1.9776	1.9954	0.9497	0.9087
-0.1518	3.7821	1.7175	1.6075
1.8860	1.9627	0.2665	-0.2232
0.5703	2.7223	1.7012	1.6334
1.7951	1.9066	2.5914	3.3946
0.9968	2.1971	1.6793	1.6567
1.7098	1.8260	1.7631	2.0231
1.3012	1.8981	1.6523	1.6758

FIVE POSITION SOLUTION - CASE 3

COORDINATES OF THE CONSTRAINT LINKS

	POINTS				CENTERS		
X =	2.00000	Y =	2.00000	X =	1.00000	Y =	1.00000
X =	1.57443	Y =	1.66542	X =	1.60472	Y =	1.66583
X =	1.81962	Y =	1.92433	X =	4.39017	Y =	6.25466
X =	1.64324	Y =	1.72212	X =	1.63001	Y =	1.75344

CHAPTER VI

RECOMMENDATIONS AND CONCLUSIONS

There are several subjects which may be logical extensions of the preceding study. Among these are the time dependent analogy of classical curvature theory and design optimization techniques which may be applied to the synthesis of multi-link mechanisms. Before concluding, preliminary formulation of the first topic will be given along with recommendations for further research on both topics.

A. Time Dependent Analogy of Classical Curvature Theory

Classical curvature theory is concerned with the specification of the instantaneous motion of a planar body for geometric properties such as tangent, curvature, rate of change of curvature and the determination from this information several special loci within the plane that may be used for application of constraints. This type of synthesis is known as infinitesimally separated position (or ISP) synthesis. Some of these special loci are, for example, the locus of all points which have zero curvature (3 ISP) so that a slider or prismatic pair constraint may be used. In addition, the locus of all points having stationary curvature (with respect to the geometric parameter θ for 4 ISP) can be obtained so that a revolute pair or pin jointed constraint link may be used.

The most definitive works on this subject have assumed $\theta \equiv t$ so all time derivatives are eliminated ($\dot{\theta} = 1$, $\ddot{\theta} = 0$, $\dddot{\theta} = 0$, $\ddddot{\theta} = 0$) in addition to the assumption of a special reference $x = y = \theta = 0$ and $\dot{x} = \dot{y} = 0$. The second condition means the first position is at the velocity pole. Also, in classical curvature theory all derivatives are geometric derivatives $x' = dx/d\theta$, etc. Here, all derivatives will be with respect to time including the derivatives of θ ($\dot{x} = dx/dt$, $\ddot{\theta} = d^2\theta/dt^2$, etc.). To allow this formulation to be simplified to a reasonable extent, the same assumption will be made by placing the first position at the origin ($\theta = x = y = 0$) but the first position will not be coincident with the velocity pole ($\dot{x} \neq 0$, $\dot{y} \neq 0$) and ($\dot{\theta} \neq 0$, $\ddot{\theta} \neq 0$, $\dddot{\theta} \neq 0$, $\ddddot{\theta} \neq 0$) will be considered. The analogous loci to some classical loci from curvature theory will be shown in addition to loci not found in this geometric based theory.

As before, the specifications are given by

$$x^{k} = \frac{d^k x}{dt^k}\ , \quad y^{k} = \frac{d^k y}{dt^k}\ , \quad \theta^{k} = \frac{d^k \theta}{dt^k}\ .$$

The fixed reference system is given by (U,V) while the reference system fixed in the moving body is (u,v). With the conditions $x = y = \theta = 0$, the coordinate transformation and time derivatives are given by:

$$\begin{aligned}
U &= u & V &= v \\
\dot{U} &= -\dot{\theta}v + \dot{x} & \dot{V} &= \dot{\theta}u + \dot{y} \\
\ddot{U} &= -\dot{\theta}^2 u - \ddot{\theta}v + \ddot{x} & \ddot{V} &= \ddot{\theta}u - \dot{\theta}^2 v + \ddot{y} \\
\dddot{U} &= -3\dot{\theta}\,\ddot{\theta}u + \dot{\theta}^3 v - \dddot{\theta}\,v + \dddot{x} & \dddot{V} &= -\dot{\theta}^3 u + \dddot{\theta}\,u - 3\dot{\theta}\,\ddot{\theta}v + \dddot{y}\ .
\end{aligned} \tag{6-1}$$

The derivatives of the path specifications $\overset{k}{(x}, \overset{k}{y})$ may be expressed in terms of the equivalent geometric derivatives and time derivatives of theta by the chain rule for derivatives to facilitate comparison with classical curvature theory loci.

Since this is a discussion of possible further study, the properties will be extended to the third order. Fourth order loci may be obtained in the same fashion. Also, for these special properties or loci to be useful to the designer using MECSYN, they must be written in terms of the coefficients $D_{m\ell}$ given in Chapter II so that they are completely general ($x \neq 0$, $y \neq 0$, $\theta \neq 0$).

Some preliminary formulas will prove valuable for finding the desired loci. After the specifications for the origin of the (u,v) system are given, $\overset{k}{(x}, \overset{k}{y}, \overset{k}{\theta})$, the path of any point (u,v) at the point (U,V) has the curvature

$$\kappa = \frac{\left[\dot{U}\ddot{V} - \ddot{U}\dot{V}\right]}{\left[\dot{U}^2 + \dot{V}^2\right]^{3/2}} \quad . \tag{6-2}$$

The acceleration of a point (u,v) at the point (U,V) moving along a path S is

$$\underline{a}(t) = \frac{d^2S}{dt^2}\,\underline{T} + \left(\frac{dS}{dt}\right)^2 \underline{N} \; . \tag{6-3}$$

where $\underline{T}$ is the unit vector tangent to the path and $\underline{N}$ is the unit vector normal to the path at (U,V) where

$$\left(\frac{dS}{dt}\right)^2 = \dot{U}^2 + \dot{V}^2 \; . \tag{6-4}$$

The jerk of a point (u,v) at (U,V) is given by

$$\underline{j}(t) = \frac{d}{dt}\underline{a}(t) = \left[\frac{d^3S}{dt^3} - \kappa^2\left(\frac{dS}{dt}\right)^3\right]\underline{T} + \left[\left(\frac{dS}{dt}\right)^2\frac{d\kappa}{dt} + 3\kappa\,\frac{dS}{dt}\,\frac{d^2S}{dt^2}\right]\underline{N} \ . \qquad (6\text{-}5)$$

The fourth derivative (jounce) may be used to find the tangential and normal loci in the same manner as for the second and third order properties. The k^{th} order motion poles may be of interest to the designer. They are defined as the points in the plane, $(\bar{u}_k,\bar{v}_k)$, which have no k^{th} order motion vector (e.g., zero velocity, zero acceleration, etc.)

$$0 = \sqrt{\overset{k}{U}{}^2 + \overset{k}{V}{}^2} \rightarrow \overset{k}{U} = 0, \quad \overset{k}{V} = 0 \ . \qquad (6\text{-}6)$$

The two poles P_1 and P_2 will be shown to illustrate the dependence of the higher poles on $\overset{k}{\theta}$.

Using the chain rule for derivatives where $g \equiv \ddot{\theta}/\dot{\theta}^2$,

$$P_1 \qquad \bar{u}_1 = -\dot{y}/\dot{\theta} = -y'$$

$$\bar{v}_1 = \dot{x}/\dot{\theta} = x' \qquad (6\text{-}7)$$

$$P_2 \qquad \bar{u}_2 = (-\dddot{y}\dot{\theta}+\dddot{x}\dot{\theta}^2)/(\dot{\theta}^4+\ddot{\theta}^2) = (-y''g \quad -y'g^2+x''+x'g)/(1+g^2)$$

$$\bar{v}_2 = (\dot{\theta}^2\dddot{y}+\ddot{\theta}\dddot{x})/(\dot{\theta}^4+\ddot{\theta}^2) \quad = (y''+y'g+x''g \quad +x'g^2)/(1+g^2) \ .$$

Note that P_1 is dependent only on the geometric derivatives, while P_2 is dependent also on $\dot{\theta}$, $\ddot{\theta}$. For the analogy of P_2 in classical curvature theory set $\dot{\theta} = 1$, $\ddot{\theta} = 0$ which gives

$$P_2 \qquad \bar{u}_2 = x'' \ , \quad \bar{v}_2 = y'' \ .$$

Cubic of Stationary Curvature

A locus of considerable value in geometric ISP synthesis is the locus of stationary curvature (4 ISP). Although MECSYN contains routines for solutions for all combinations of 4 FSP and 4 ISP (4 MSP) for both time dependent and geometric specifications, this locus will be given here to show its relation to the other special loci under consideration. The problem can be stated as:

Given a set of specifications for the third order motion of a body $(x,y,\theta)_k$ $_k$ $_k$, find the locus of all points (u,v) which have at the points (U,V), stationary curvature (with respect to time).

Since the condition of stationary curvature means the points (u,v) have third order contact with a circle, these points may be used as pin joints of circular constraint links for the motion as specified. In classical curvature theory, this locus is expressed in terms of geometric derivatives only. It will be shown (as in Chapter II) that this locus is not sufficient to synthesize a mechanism for third order time derivatives. The additional condition will be shown to be related to the tangential jerk component. The constraint equation formulation in Chapter II shows the necessity of this extra condition for all cases of time dependent MSP.

The locus of stationary curvature is obtained by setting the time derivative of Eq. (6-2) equal to zero.

$$\frac{d\kappa}{dt} = \frac{\left[\dot{U}^2+\dot{V}^2\right]\left[\dot{U}\dddot{V}-\dddot{U}\dot{V}\right] - 3\left[\dot{U}\ddot{V}-\ddot{U}\dot{V}\right]\left[\dot{U}\ddot{U}+\dot{V}\ddot{V}\right]}{\left[\dot{U}^2 + \dot{V}^2\right]^{5/2}} = 0 \qquad (6\text{-}8)$$

or

$$\left[\dot{U}^2+\dot{V}^2\right]\left[\dot{U}\,\dddot{V}-\dddot{U}\,\dot{V}\right]-3\left[\dot{U}\ddot{V}-\ddot{U}\dot{V}\right]\left[\dot{U}\ddot{U}+\dot{V}\ddot{V}\right]=0 \qquad (6\text{-}9)$$

$$\text{if } \dot{U}^2+\dot{V}^2\neq 0\ . \qquad (6\text{-}10)$$

Equation (6-9) appears to be fourth order in (u,v) but the analogy in classical curvature theory has shown this to be cubic and is called the cubic of stationary curvature. The condition, inequality (6-10), will be necessary for some of the following loci. With the usual assumption that the origin of (u,v) is placed at P_1, this condition reduces to $u \neq 0$, $v \neq 0$. However, it represents, in general, the exclusion of all points (u,v) which lie at the intersection of the locus in question and a circle represented by inequality (6-10);

$$(u^2+v^2)+2(\dot{y}/\dot{\theta})u-2(\dot{x}/\dot{\theta})v+(\dot{x}/\dot{\theta})^2+(\dot{y}/\dot{\theta})^2\neq 0$$

or

$$(u^2+v^2)+2y'u-2x'v+x'^2+y'^2\neq 0\ . \qquad (6\text{-}11)$$

For all the loci to which this condition pertains, one solution is the pole $P_1 = (-y',x')$.

Inflection Circle

The locus of all points (u,v) having a characteristic curvature, κ, is

$$\left[\dot{U}\ddot{V}-\ddot{U}\dot{V}\right]-\kappa\left[\dot{U}^2+\dot{V}^2\right]^{3/2}=0\ . \qquad (6\text{-}12)$$

This is sixth order, in general. For $\kappa = 0$ (useful for 3 ISP sliding constraint) then

$$\dot{U}\ddot{V}-\ddot{U}\dot{V}=0 \quad \text{if} \quad \dot{U}^2+\dot{V}^2\neq 0\ . \qquad (6\text{-}13)$$

Note that this is given by the third term of the cubic of stationary curvature and is referred to as the inflection circle. Any point (u,v) on this circle undergoes an inflection for the given motion specifications. Using the chain rule, the inflection circle can be written in terms of geometric derivatives only, as

$$u^2+v^2 + (y'-x'')u + (x'-y'')v + x'y'' - x''y' = 0 \qquad (6\text{-}14)$$

with center at

$$[(-y'+x'')/2,\ (-x'+y'')/2]$$

and radius

$$[x''y'-x'y'' + (y'-x'')^2/4 + (x'-y'')^2/4]^{\frac{1}{2}} \ .$$

Zero Tangential Acceleration Circle

From Eq. (6-3), the tangential acceleration is

$$a_t = d^2S/dt^2 \ .$$

The locus of all points (u,v) having a characteristic tangential acceleration, a_t, is

$$[\dot{U}\ddot{U} + \dot{V}\ddot{V}] - a_t[\dot{U}^2 + \dot{V}^2]^{\frac{1}{2}} = 0 \ . \qquad (6\text{-}15)$$

This is fourth order, in general. For $a_t = 0$, the zero tangential acceleration circle arises.

$$\dot{U}\dddot{U} + \dot{V}\dddot{V} = 0 \quad \text{if } \dot{U}^2 + \dot{V}^2 \neq 0. \qquad (6\text{-}16)$$

Note that this is given by the fourth term of the cubic of stationary curvature. Equation (6-16) may be written

$$g(u^2+v^2) + (y''+2gy'-x')u - (x''+2gx'+y')v$$

$$+ x'x'' + y'y'' + g(x'^2+y'^2) = 0 \qquad (6\text{-}17)$$

where

$$g \equiv \ddot{\theta}/\dot{\theta}^2 \ .$$

This shows that Eq. (6-16) is dependent on the time functions $\dot{\theta}$ and $\ddot{\theta}$. The analogy to Eq. (6-17) in classical curvature theory may be found by setting $\dot{\theta} = 1$, $\ddot{\theta} = 0$, giving

$$(y''-x')u - (x''+y')v + x'x'' + y'y'' = 0 \ . \qquad (6\text{-}18)$$

This is the equation of the normal to the inflection circle passing through the pole $P_1 = (-y',x')$.

Zero Normal Acceleration Circle

The normal acceleration is given by Eq. (6-3) as

$$a_n = \kappa\left(\frac{dS}{dt}\right)^2 \ .$$

The locus of all points (u,v) having a characteristic normal acceleration, a_n, is

$$[\dot{U}\dot{V} - \ddot{U}\dot{V}] - a_n[\dot{U}^2 + \dot{V}^2]^{\frac{1}{2}} = 0 \ . \qquad (6\text{-}19)$$

For $a_n = 0$ this locus is identical to the inflection circle. However, for $a_n \neq 0$ the two loci are distinct. Recall the characteristic curvature locus is

$$[\dot{U}\dot{V} - \ddot{U}\dot{V}] - \kappa[\dot{U}^2 + \dot{V}^2]^{3/2} = 0$$

which is sixth order, while Eq. (6-19) is at most fourth order.

Zero Normal Jerk Circle

The normal jerk is given by Eq. (6-5) as

$$j_n = \left(\frac{dS}{dt}\right)^2 \frac{d\kappa}{dt} + 3\kappa \frac{dS}{dt} \frac{d^2S}{dt^2}$$

but

$$a_n = \kappa\left(\frac{dS}{dt}\right)^2 \quad \text{and} \quad \frac{da_n}{dt} = \frac{d\kappa}{dt}\left(\frac{dS}{dt}\right)^2 + 2\kappa \frac{dS}{dt} \frac{d^2S}{dt^2} \ .$$

So j_n may be written

$$j_n = \frac{da_n}{dt} + \kappa \frac{dS}{dt} \frac{d^2S}{dt^2} \ .$$

From the above it can be shown that the locus of all points (u,v) which have a characteristic normal jerk, j_n, is

$$[\dot{U}\,\dddot{V} - \ddot{U}\,\ddot{V}] - j_n[\dot{U}^2 + \dot{V}^2]^{\frac{1}{2}} = 0 \ . \tag{6-20}$$

For $j_n = 0$, the zero normal jerk circle arises

$$\dot{U}\,\dddot{V} - \ddot{U}\,\ddot{V} = 0 \quad \text{if} \quad \dot{U}^2 + \dot{V}^2 \neq 0 \ . \tag{6-21}$$

This equation is represented by the second term of the cubic of stationary curvature.

Zero Tangential Jerk Circle

The tangential jerk is given by Eq. (6-5) as,

$$j_t = \frac{d^3S}{dt^3} - \kappa^2\left(\frac{dS}{dt}\right)^3 \ .$$

The locus of all points (u,v) which have a characteristic tangential jerk, j_t, is

$$[\dot{U}\,\dddot{U} + \dot{V}\,\dddot{V}] - j_t[\dot{U}^2 + \dot{V}^2]^{\frac{1}{2}} = 0 \ . \tag{6-22}$$

When $j_t = 0$, Eq. (6-22) gives the zero tangential jerk circle.

$$\dot{U}\,\dddot{U} + \dot{V}\,\dddot{V} = 0 \quad \text{if } \dot{U}^2 + \dot{V}^2 \neq 0 \ . \tag{6-23}$$

Note that this equation is not represented in the cubic of stationary curvature. For a geometric problem, it would not be necessary, but for a problem involving specifications which are a function of time this equation represents the essential tangential properties of the constraint circle point moving along the path. Thus the second condition for synthesis on page 29 is the same as the inclusion of this condition. Both signify the need for a synthesized input which effectively guarantees the constraint point or circle point will have the necessary tangential motion properties needed to satisfy the specifications.

Other loci such as the characteristic normal and tangential jounce loci may be found by differentiating Eq. (6-5) with respect to time. An important point to the designer is Ball's point. In classical curvature theory, Ball's point is a point where the curvature of the path has the stationary value zero or third order contact with a straight line. This point may be used for a 4 ISP linear or sliding constraint similar to the 3 ISP sliding constraints of the inflection circle. The analogy of this point may be found by the intersection of the zero normal acceleration circle (inflection circle) and the zero

normal jerk circle. Also it may be of interest to investigate the meaning of the intersection of other loci of characteristic accelerations and jerks.

The characteristic acceleration and jerk loci appear to be fourth order, while the zero acceleration and jerk loci are all second order. It may be of interest to investigate the nature of these characteristic loci and their possible use to aid the designer. The zero loci pertain to two common types of constraints, i.e., circular and linear, while the characteristic loci may be used to describe any type of general constraint.

B. Mechanism Evaluation for Design Optimization

The term optimization used in the context of mechanism design does not refer to classical optimization methods such as those based on calculus of variations but rather to the determination of effective, easily computed criteria by which synthesized mechanisms may be rejected or accepted. There are exceptions to this, such as optimal precision point spacing for function generation using Chebyshev polynomials, a method well known to kinematicians. However, this is an optimization method that is applied before the synthesis of the mechanism.

Although the synthesis process gives the designer great freedom in the choice of motion specifications, frequently the resulting mechanism must also satisfy other conditions such as force transmission, mobility and size requirements. Some of these criteria are time consuming to calculate, especially when many mechanism solutions can be generated for the same specifications by variation of a parameter (as with

most six-link mechanisms), or by relaxing the specifications (four-position problem). Accordingly, some of these criteria have been included in MECSYN. Other criteria are not well developed, e.g., general mobility criteria for six-link mechanisms similar to those developed by Grashof for a four-link chain. Some of these criteria may be determined by a complete motion analysis of the synthesized mechanisms, a very undesirable procedure considering computer time and necessary core storage.

After all the necessary criteria have been formulated and included in MECSYN, the designer may wish to compare these criteria personally or in the event of multiple solutions a cost function or penalty function may be used to automatically eliminate mechanisms. However, most mechanism applications require different criteria for design. Thus, any reasonable scheme for a penalty function must allow the designer to input weight factors for each criterion. A short description of several design criteria of importance in multi-link synthesis follows.

A most obvious condition for design is the relative size of a mechanism. Computation of the link lengths and link length ratios (ratio of longest to shortest link) tells the designer if the mechanism fits in the allowable space and if the mechanism can be effectively constructed (very small links may create manufacturing problems). MECSYN computes link lengths, link length ratios and modified link length ratios for all four-link modules and their cognates. This may easily be extended to compute link length ratios for a complete multi-link mechanism. The modified link length ratio uses an equivalent

link length for a ternary coupler link. The average of the two rays extending from the pin joints to the coupler point is used as an additional link in the link length ratio calculation.

A useful measure of mobility of a four-bar mechanism is Grashof's criteria which give several rules for the determination of how the input and output cranks rotate from the mechanism proportions. This allows immediate evaluation of whether or not a mechanism may have a continuously rotating input crank, for example. These criteria are evaluated for all four-bar mechanisms and their cognates in MECSYN. At present, this can be accomplished for six-link mechanisms only by a complete motion analysis. Criteria similar to Grashof's for six-link mechanisms would be the most desirable way of providing this mobility information. An additional problem arises in six-link mechanisms for which the input is a ternary link. Frequently, although the input may rotate full cycle in the course of the mechanism's motion, this input rotation is not monotonic increasing but requires several reversals. Since this would not allow practical implementation of the mechanism, good mobility criteria should detect this condition.

A very important criterion for evaluation of mechanisms is the force transmission property of the mechanism; i.e., what portion of the force at the input is transmitted to the output. In a four-bar mechanism, this property is clearly defined by the transmission angle (angle between the coupler and output link). For some multi-link mechanisms it is more difficult to find, and, in general, can be determined by calculating the influence coefficients or velocity ratios between the input and output links.

Although the two or three modules comprising a five- or six-link mechanism are guaranteed to be compatible for the precision points specified by the synthesis process, occasionally they are not compatible for the full motion of one module. Again, this could be determined by a coupler curve analysis of one portion of the mechanism and a point-by-point check on the remaining portions of the mechanism to assure they will extend to these limits. For many mechanisms, as may arise from a four-position problem, this could be expensive in terms of computer time. This might be avoided by an efficient method of determining coupler curve bounds without actually computing the coupler curve. Then dyads, for example, may be checked against these bounds to determine if they are compatible for the complete cycle.

A similar problem arises when a dyad needs to be in different closures for different, finitely separated positions. The MECSYN analysis of dyads at precision conditions gives the closure a dyad must be in to satisfy the specifications. Determination of the dyad closures for a series of points on a coupler curve between the specified finitely separated positions will indicate whether or not the dyad is in the correct closures. If a four-bar module of a multi-link mechanism satisfies the specifications in two different closures, it can be immediatly discarded. This type of closure test would be useful but is not provided by MECSYN at present.

Another similar problem is that of the order in which finitely separated position specifications are satisfied. This is not a frequent problem, if the specifications are chosen to give a smooth progression of finitely separated position.

The preceding two design difficulties are less likely to arise when higher order properties are specified. They occur mainly when many finitely separated positions are specified.

When a function or functions are approximated by several precision conditions as in the gear cutter example of Chapter IV, it is important to know the error, that is, the difference between the desired functions and the generated functions. This requires an analysis of the synthesized mechanism for the desired functions.

By the methods presented here, the complete motion of a body may be coordinated with an input. Thus an angular function and a path function are both generated. The well-known method of optimal precision point spacing by Chebyshev polynomials should have an analogy for optimal precision position respacing for synthesis of path and angular function generators. A theory such as this would be very useful in problems such as the gear cutter mechanism of Chapter IV.

If all the preceding suggestions are implemented in MECSYN, the designer's task would be greatly reduced, leaving only the problem of choosing the desired specifications and design criteria.

C. Conclusion

The theory and design methods presented herein allow the synthesis of a large class of mechanisms for more realistic problem specifications than available with geometric synthesis techniques. The author's previous work in five- and six-link mechanism synthesis methods has been extended to include many previously unsolved geometric synthesis problems.

All the synthesis problems and all cases of mixed specifications presented can be solved by the computer program MESCYN in addition to other problems not treated here.

MECSYN is a completely closed form, noniterative, inexpensive program that may be used on most computers. MECSYN can be used effectively with minor knowledge of the underlying theories.

Hopefully, the methods and computational tool presented here will encourage the solution of practical problems by mechanism synthesis.

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BIOGRAPHICAL SKETCH

Arvid Myklebust was born August 16, 1945, in Bergen, Norway. He attended primary school in Miami, Florida, and was graduated from North Miami Senior High School in June, 1963. He attended Miami-Dade Junior College evening classes from 1963 to 1967 and received the Associate of Arts degree. He transferred to the University of Florida in September, 1967, and received the Bachelor of Science in Mechanical Engineering degree with high honors in December, 1969. He began graduate studies in Mechanical Engineering, January, 1970, funded by a College of Engineering Fellowship and later a National Science Foundation Traineeship. He received the Master of Science in Engineering degree in August, 1971.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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